'Exceptions' in Queuing Theory

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Abstract—**The efficiency of queuing system is depends upon the behavior of customer(s) and server(s). Most of time the problem in queuing system arises due to some unethical behaviors. An analysis and approximation of queuing models with specific behaviors during various situations has been covered by many researchers. However, many obscure behaviours factors can be appealing to queuing models, which are still at large to consider during the study of such system. There are occasions where ignorance due to cultural background can block the queue flow. Under those circumstances the 'exception' occurs. This paper shall introduce some of behavioural events that are not in compliance with study of queuing system rules, are termed as 'exceptions'. In order to model how the 'exception' can overcrowd and extends waiting times, the probability theory and stochastic process are a part of interesting concepts. A general purpose probability models formulated to simulates such parameters by considering certain 'exceptions', that finds out, the 'exceptions' has major affects on queuing system.**

***Keywords-Behaviour; Waiting time; Overcrowding; Erlang; Exception; Non-queue; Delays in queuing system*.**

I. INTRODUCTION

If the queuing system considers human involvements then most probably the only term relevant is the waiting. Besides this, the system in services, even though not desirable allows the customer to wait due to their performance rate but when exception comes in current task the waiting time becomes longer. It possibly will be an undesirable exception caused either the service provider gets undesirably slow or service seekers are undesirably arriving faster than normal. The sudden changes in behaviours of either customer(s) or server will result in an unbalanced system. From [2] if such behaviours are not controlled in time the chances are that the queues get bigger and bigger. The balanced queuing system can only possible if contributions by seekers and host(s) are follow ethics and are organized. The services of the customer may be either constant or stochastic, and the organization of queuing system varies globally. However, in some area of world the queue routines are not following properly. From place to place the cultural practice of queuing includes awful customers, dishonor and for few the existence of queue is almost alien to them. Such practices cause pressure on the serving system.

Numerous queuing literatures discuss two types of behaviours concerned with customer and servers which are *impatience* and *vacation period*, respectively. In customer behaviours case reneging, preemptive, balking and jockeying are covered largely by other researchers. Even these events will have different classification of various appearances but the consequence affect of such undesired events overcrowd and converging to broaden the waiting time inside the queuing system. But this can ground by both customer(s) and server. Fundamentally, both the services interruption and conflicts among customers can affect customers waiting time. This affect can derive the customer impatience to bust the queue. Besides that the assumptions in queuing models are based on customer or system deterministic behavior to get a stationary state for future prediction. For instance, using M/M/1:N model force balking option in actual world could turn back the customer to never return, where the actual world is not stationary and the existence of 'exceptions' possibly will direct the serving system to a non-queue state. However, the strict policy can control customer behavior. The disruptions in routine are caused by the existence of 'exception' in the serving system. The 'exceptions' has a capability to absorb the queuing system to make it passive. Reference [4] describes the behavior of customers with two characteristic features, viz. the arrival process and the waiting process, and thus the continuation of unnoticeable behaviour may challenge the service seekers patience. References [9] and [2] says that the study of behavioral problems in queuing system is intended to understand how they behave under various conditions and when the system is $\rho \geq 1$, then they are in a saturated region and the stationary results cannot apply.

The unexpected growth of arrival rate, bulk arrival, behaviours like customer renegade and balking inside the system are noticed in many queuing literatures. The above behaviours are considered as mortifying system performances which influences customers waiting time. An article on uncertainty in arrival rate by [8], evaluated the arrival rate for inbound calls center operations. Their model simulates the impact of arrival rates on the staff plan changing for call centers. However, during the uncertainty of arrival if the customer wishes to stay longer their service cannot be fulfilled because of call hang ups distribution and eventually calls would end-up if no acknowledgement at all by the server. Thus, the waiting time distribution is not been discussed during the hang ups period. Reference [10] has analyzed the system of computer farm or a call center with M/M/c which suffers disastrous outcome that cause the loss of all running, waiting sessions and the arrival of impatient customers. However, the reason of disastrous outcome has not been discussed and the impatient customers abandon the system and never to return. Reference [1] has modelled the behaviour of customers for system where the customer performs self-service through kiosk to reduce the load on the server. However, the model analyzed the waiting time during the advancement of services and not on queuing system parameters but still depends upon customer decisions on service choice making self-service alternatives highly cost effective. Similarly, [3] analyzed the waiting time with similar conditions in addition to multiple vacations but with impatience such as balking.

Another article [5] had surveyed the review on impatient customers that cause changes in ordinary queuing system. It also stated that the other researchers ignored the impatience factor while studying queuing system. This article mentioned in their reference source that some reviews provide detailed areas in queuing model, for example, the majors research areas on the approximation techniques for solution of queue; on cyclic queues and closed queue networks; on the matrix analytic methods in queuing theory; on retrial queues; on open queuing network models of manufacturing systems; and on the matrix analytic method and working vacation queues. The article further claimed that they do not find the review work on queuing system with impatient customers although it has been widely studied in queuing literature in recent decades.

Therefore, all the above mentioned research topics have been fully explored except for two behaviours, renegade and balking to analyse the system for waiting distributions. Thus, the main role of this paper is to introduce the others ignored behaviours in the queuing literatures termed as 'exceptions' with waiting time distribution during 'exception'. Based on the observation, some of the other queuing behaviours are colonizing, splitting up strategy, underpinning, and brazenness which will affect the performance of the serving system. This paper will traverse some sort of 'exceptions' that cause serving system deflection. This paper assumes that customer impatience will not quit the queuing system. In the following, the singular phrase with quotes 'exception' will signify some obscure behavioural term associated to either customer(s) or server, unless otherwise for classifications the term 'exceptions' will used.

This paper has organized into four main sections. Sections II introduce and classify the types of 'exceptions' in queuing system in other subsections with examples. Followed by Section III that formulates mathematical models and simulates the waiting time distribution during specific 'exception' and the conclusion is in the subsequent Section IV.

II. Categorization and Advanced Classification of 'Exceptions'

This section has categorized 'exceptions' in two ways, a quantity of 'exceptions' occurs before the formation of queue and some occurs during the normal queue flow within the system. The advance classifications of 'exceptions' are *changes in server approach, changes in system policy* and *changes in customer behavior*. However, either of the 'exceptions' classification can cause a decrease in service performance levels followed by large growth of customers.

A. Queuing System with Server Disreputable Approach

Some serving systems due to their lower maintenance, fluctuate or rare equipments and software update may not be synchronized by global changes. By this, the additional chances of dissatisfactions by the customers may balk the system for another or may lose their patience while waiting for their services. Therefore, some of the common 'exceptions' discussed below are caused by the server.

1) Bypass serving system: When the server is biased to a particular group of customers it will cause impatience in other customers and may push the system to a non-queue state. For e.g., some server serves specific customer(s) known to the server. In single server system the server has accepted the direct access for services without any respect to queuing customers, indeed an 'exception'. Hence, such behaviour will extend the length of waiting time. Then, if Y_e the waiting time random variable for a particular service that exceeds the mean waiting time t given as $P\{Y_e > t\} = e^{-\mu t}$. Surely in such cases, the mean waiting time for each customer will now equal to the average waiting time in addition to the mean service time of bypassed customer(s). Therefore, the probability of total waiting time will be $P\{Y_e > t_i + t_j\}$, where t_i is the mean waiting time of the i^{th} customer and t_j is j^{th} bypassed customer(s) mean service time. If the waiting customer is at n_i^{th} position, then due to k interferences, the i^{th} customer position by the next time would be shifted back to $(n+k)_i^{th}$ position. Therefore, the probability of waiting time of i^{th} customer will be defined by $(t_i + kt_j)$. In this 'exception' the analyses of waiting times is needed.

2) Delay commencing of services: This category of 'exception' will occur both before and after the queue formation. In this case, when customers are already present in the system with or without formal queue and yet system has not started providing services yet. This general form of 'exception' involves the excessive urge to collect extra customers or open house events, for e.g., the private transporting providers, clinics, ticket concert sellers, late arrival of artist or politicians, ration distributors and others. If the queue exists, then the customers total waiting time will be depends upon the waiting before (t_b) for the initiation of the services and the waiting time (t_a) after server start until they get their services. Moreover, when services are already delayed, the customers may be inclined to place themselves without formal queue. If there is no formal queue caused by system delaying the opening, it may initiate as non-queue state. The waiting time distribution evaluation is needed whereas for latter case the study of queue formation and queuing parameters are analysed.

3) Tailored waiting: This 'exception' occurs when some customer(s) are given preference over already in service for "at queue head", requiring direct service. Even if the queue discipline is first-in first-out (FIFO) but in this case there could be incomplete last-in first-out (LIFO), i.e. favoritism to last-in or queue less customer(s). The typical scenarios are, when a request for service from recognized customer, backend pressure for immediate service, unequal service ethics, emergency services and bias to certain group or individuals. If the customers at queue head are being tailored waiting for number of times due to preferential services, then this 'exception' may challenge the patience of immediate affected customer as well others to dissolve the queue. The waiting time distribution as the customer becomes impatient till back in service assessments are essential.

B. Queuing System with Unsystematic and Fluctuate Policy

Some serving systems are practicing unsystematic plans with policy fluctuation. The frequent types of procedures associated with private interests, relevant to safety, government services and human resources. A sudden change in private interests and government policies will highly affects the customers receiving services by queuing. The policy fluctuation 'exception' exhibits the attitude of unsystematic plans and this affects wherever waiting in queue is engaged. Thus, the following are some 'exceptions' under the unorganized plans that cause lengthier waiting time, if they occur.

1) Devices unconcern system: This 'exception' category occurs within the queuing system. The overcrowding in the queuing system by this 'exception' normally happens when the server is unable to handled peak hour loads due to ineffective output by either hardware or software. It occurs by poor devices maintenance within the defined policy. Some of the examples are old version software usage and hardware wear & tear. However, if the customer arrival is still being received continuously by the system, then it can reach a collapsing situation. In this 'exception', if resuming of queue flow is not possible then the system requires reformatting, and then fresh queuing system parameters analyses expected.

2) Less informative system: This sort of 'exception' can occurs before or after queue formation. In some cases, the serving systems are mostly unorganized in policy distribution that stretches the customers waiting time. The typical plots are selling products and special purpose registration counters that should indicate brief required information for naive customer(s). For example, the information resembling which counter to proceed for what purpose, keep enough currency change, general services average waiting time and others. Therefore, the less information provided through system certainly will cause overcrowding of customers inside the system.

3) Policy fluctuate system: This 'exception' generally occurs inside the queuing system when the queue is active. The frequent plan changes are mostly seen for customers waiting for public/private transport such as buses, train services and airplanes. The frequent changes in schedules prolong in waiting time. Therefore, the frequent changes will surely overcrowd the system. It is expected to measure the comparison of waiting time before and after this 'exception' until the departure.

4) Broadcasted appointments: In this common form of 'exception' which can observed mostly in public clinic wards as well as sometimes during interview sessions for various tasks. In the clinic scenarios visiting, special noted patients after treatment will be called for the next visit but normally at the same time. Due to appointments broadcasted at the same time will cause the early arrival of clients at the same time. Moreover, before the services begin, large number of clients are accumulated at service window and possibly will cause rushing to get the queue numbers for their turns. Even the service mean rate may not be affected but there will be no queue and the clients impatiently waiting for their turns. As a result, if there is already a queue then the analyses of t_b and t_a are required else if not a queue at all then non-queue transformation expected and rest valuations. Although the server is part of system policy, the changes in policy shapes the changes in server dealings. However, under such cases, it show differences when the server independently behaves not in favor of queuing ethics. Moreover, this generally occurs if server is controlled by human with misleading behaviour. Hence, whether the disruption caused by the server or by the whole system, fluctuation in any case affects the whole queuing system.

C. Queuing System with Changes in Arrival

In this 'exceptions' the behaviour of customer results in the enhancement of waiting times in queue as well in the system. In this case, servers are generally independent of issues faced by the customers. If the 'exceptions' mentioned in the previous section are not valid for discussion then the sudden changes in behaviour of customers still exists. The greater part of times when any buying or selling processes are in large then the behaviour of customers are not in ethics. Some of the common behaviours are balking and jockeying highly been discussed in queuing literatures. Apart from these, the following common behaviours can be observed in any queuing systems.

1) Inconsiderate underpinning: This 'exception' happens when the customers are already in the queuing system, but has not wished to join the queue. These non-queued (queue less) customers are expecting the server adjoining to provide services. Some of the common examples mostly can be discovered at infrequent transport ticketing counters, theater/amusement ticket counters, shopping complex counters and immigration centers. Even non-queue customer(s) are not allowed, but if the k outside customer(s) underpinned by i^{th} queued customer then $(n-i)$ customers waiting times will then depends upon the $(k+i)$ interposed customers. Therefore, the requirement for analyses of waiting time will be the same as the situation wherever the server bypasses individual or specific group.

2) Brazenness feeding: The typical illustration of this 'exception' is opposite to the server biasness. In this case, the front-end pressure derived by outsider(s) is expected to receive prompt service regardless of any queuing ethics. This 'exception' normally occurs within the system when any personal authorization abuser or a group forcibly interrupts in the queue. It could represent any individual or group that has inconsiderably violated queue morals to get quick services at the overhead of others. This will be surely affects the later customers. However, in this situation the waiting time will be similar to *bypassing* or *underpinning*.

3) Unorganized customer: Similar to the system providing misinformation, sometimes an uncoordinated customer can turn into 'exception' to slow down the service process. This 'exception' generally arises while a customer is not organized to fulfill any demand arising from server. In this case, unorganized customer(s) take up the other queued customers mean service times. Generally, rather than while in queue, most of the customers construct their decisions only at counters, even if the system has provided enough information to minimize the waiting time. For e.g., fast food decision only at counter, asking useless information at ticket/selling counters, never keeps currency change, unprepared with documents at special purpose registration counters. The major form of 'exception' occurs at study registration counters and airports where due to inattentiveness of many clients/customers to produce or carrying related documents during common enquiries. In another view, if this 'exception' persists then the chances that customers may move out of the formal queue and take up random positions in the system, and then non-queue state begins. Consequently, the waiting time study are required during both queued or system reformation.

4) Untimely arrival: This 'exception' is categorized as one that occurs before queue formation. This situation happens during sudden sale promotions publicized by the serving system for a limited time. Even this sort of policies can result in system with crowd but the system may not be expecting that the customers lose their ethics and begin to crowd. In this state, there will be sudden buildup of early customer arrival when the outlet has not commenced yet and followed by either queue formation or no queue at all. In this 'exception' the analyses of t_a, t_b and service rate are necessarily required. Therefore, by seeing the affect of occurring of any such 'exception' the knowledge of unpredictable behaviour of seeker/host is considered as unknown to each other.

In the following section only few common 'exception' events used for waiting time distribution assessment. Usually, the waiting happens earlier prior to system initiation, inside system during customer(s) inconsideration and inside the system during server inconsideration. For waiting time assessment purpose, this paper considers the first event is the waiting before and after the system commencement shall be compared with different sub-cases. While in the second event, the customers waiting time shall be evaluated when interposing act alters their positions in queue. In other events, the waiting time affected when server alters service rate shall be evaluated. Following subsection *A* and *B* will assume theoretical distribution i.e. exponential distribution for the waiting time and as well as for the service time.

III. Waiting Time Distribution Evaluation

A. Untimely Arrivals Waiting Time Distribution

In this 'exception' when the customers are early on arrival if there are some sales or free services offered for limited period. The beginning of services shall be considered as queuing system initiation or vice-versa. The two scenarios, before and after the service begins until the customer departure will be discussed here. In the first scenario when an arrived customer(s) are unfamiliar with the beginning of service time and in the second scenario when arrived customers knows the opening instant of system. In both scenarios, further two sub-cases can imagine; when customers arrived early before the system begins as either all are already in the queue or in the non-queue state. Furthermore, before service commences, the system services are independent of how the customers arrived. If only, all customers are already in queue or a Poisson arrival. Since the service time inside the system are not concerned with the arrivals as the distribution of arrivals only matters when evaluation for cumulative waiting distribution in queue as well as the waiting time in whole system. For the first sub-case, only queued customers will be discussed. In each scenario, the customers can arrive either in group or individually.

1) First scenario as unknown opening time: This case taken before the system begins and the customer does not know when it will start. The estimation rate of that λ_1 tells the distribution of waiting time before system commence. The best examples can be concert tickets selling, waiting for artist or some politicians. The following section will evaluate the case when customers are already in queue before the system begins.

a) Earlier queuing waiting time distribution: For this type of 'exception' the arrival is considered to have a Poisson or other distribution model when the queuing system begins. If the customer arrived in group then it will be considered as an arrival of individual because all are sharing the same distribution. Then, for the each individual arrived at continuous time $T_1 < \ldots < T_i < \ldots$, where T_1 arrived earlier than T_2 and so on. Then, $\mathcal{T}_{b(1)}$ is the waiting time before the outlet opens for customer to arrive at T_1, where t is the estimated opening time of the outlet, then the evaluation shall be $P\left(\mathcal{T}_{b(1)} > t\right)$, where $\mathcal{T}_{b(1)} > \ldots > \mathcal{T}_{b(i)} > \ldots, t \in \mathbb{Z}_+$. As soon as i^{th} customer arrived at T_i then their waiting time probability will be more than t before outlet open is $P(\mathcal{T}_{b(i)} > t)$, then for the $(i+1)^{th}$ customer arrived at T_{i+k} the probability is. $P(\mathcal{T}_{b(i+1)} > t)$.

The individual waiting time for any t is independent of each other and each customer will have different estimation for different values of t. If the customer arrived at $\mathcal{T}_{(i+k)}$ then their $\mathcal{T}_{b(i+k)}$ is independent of customer arrived at T_i and the distribution for customers waiting times from $\mathcal{T}_{b(i)}$ and $\mathcal{T}_{b(i+1)}$ shall be same. As T_i arrived earlier than all, followed by the probability of $\mathcal{T}_{b(i)}$ with Markov property of $P\left(\mathcal{T}_{b(i)} > t + x \,|\, \mathcal{T}_{b(i)} > t\right)$. Therefore, it implies the probability that customer arrived T_i will wait at least $t + x$ units given that it has already waited t units, then it is the same as the initial probability that the customer has waited for at least t units. Then, the condition $P\left(\mathcal{T}_{b(i)} > t+x \mid \mathcal{T}_{b(i)} > t\right) = P\left(\mathcal{T}_{b(i)} > x\right) \forall t,x \geq 0$ is same as

$$P\left(\mathcal{T}_{b(i)} > t+x,\, \mathcal{T}_{b(i)} > t\right) / P\left(\mathcal{T}_{b(i)} > t\right) = P\left(\mathcal{T}_{b(i)} > x\right) \quad (1)$$

By Equation (1) with $\mathcal{T}_{b(i)} \sim \exp(\lambda_1)$ and λ_1 the estimation parameter of before time waiting, then the conditional probability for customer arrived at T_i is $e^{-\lambda_1 x}$. The customer arrived at T_i has conditional distribution but for customer arrived at T_{i+k}, as well, the same distribution as $e^{-\lambda_1 x}$. Hence, Equations (1) is same as $e^{-\lambda_1 x}$, therefore the distribution of current and previous customer is

$$P\left(\mathcal{T}_{b(i+1)} > x\right) = P\left(\mathcal{T}_{b(i)} > t+x,\, \mathcal{T}_{b(i)} > t\right) \quad (2)$$

If Equation (2) is successively arrived customer distribution, then this distribution will even be same for those customer(s) who arrived much later than that of earlier arrived customer(s). Equation (2) shows that $\forall \mathcal{T}_{b(*)}$ distribution is memoryless and independent because the waiting duration of any customer arrived at any time is resourced by the queuing system commencement but not the arrival of previous customer(s). Equation (2) clearly depicts that all individual have the same distribution but the actual proportion of time spent before system begins possibly will differ.

b) Later queuing waiting time distribution: The waiting time distribution after the system begins will be the same for each customer only if each have parallel services. Since the server is single, thus, the waiting time spent before the system begins is not regarded by the system own time distribution assessments. Other than that, customers are only concerned with their total waiting from early arrival until departure. For all, $\mathcal{T}_{a(*)}$ is the waiting time set after the queuing system begins or the proportion of time spent in the system. When the service begins then each customer will withdraw latest mean service time from service rate distribution and each customer waits for the service. Regardless of waiting time distribution before queuing system begins, the service is independent and identically distributed with mean λ_2^{-1}.

Then, the $\mathcal{T}_{a(1)}$ the random variable for first customer at the counter and the probability estimation that it will take z for the service completion is. $P\left(\mathcal{T}_{a(1)} > z\right) = e^{\lambda_2 z}$, $z > 0$. As a result, according to latest service rate (λ_2), time spent in waiting by first customer is same as the time spent for the service because service begins from it. For overall case, the first customer waiting earlier before system opens until the departure is $P\left(\mathcal{T}_{a(1)} > \mathcal{T}_{b(1)} + z \mid \mathcal{T}_{a(1)} > \mathcal{T}_{b(1)}\right) = \frac{f\left(\mathcal{T}_{b(1)}+z\right)}{f\left(\mathcal{T}_{b(1)}\right)}$, where in conjunction with $\mathcal{T}_{a(1)}$, the $\mathcal{T}_{b(1)}$, is summarized times of waiting during earlier queuing. Then, by $\sim\exp(\lambda_1,\lambda_2)$ and solving the distribution is the same with different parameter as in Equations (1) and when $e^{-\lambda_1 x}$, which gives $e^{-\lambda_2 z}$, $z > 0$. Therefore, from Equation (2) if both times set with parameter λ_1,λ_2 are seen as the first customer at counter then waiting distribution before queuing system begins and until departure are distributed exponentially, then $\mathcal{T}_{b(1)} \sim \exp(\lambda_1) = \mathcal{T}_{a(1)} \sim \exp(\lambda_2)$. The above expression only entails that both waiting distribution sets are exponentially distributed but not with the same proportions. The proportion of time spent after the system begins might be less than that spent earlier. As the $\sim\exp(\lambda_1,\lambda_2)$ does not relate with system view for waiting time distribution, then overall waiting distribution view in the sense of customer will be then based on joint distribution of λ_1 and λ_2, where λ_1 and λ_2 occurring in sequence and $\lambda_1 > \lambda_2$. If $P(\mathcal{T}_{a(1)} > z) = e^{-\lambda_2 z}$, then if j^{th}, customer later than $\mathcal{T}_{a(1)}$ is already waiting for t units and for j^{th}, it will take $T = \left(t + y_{j-1}\right), y_{j-1}$ is the sum of $(j-1)$ customers, as waiting time in system due to previous $j-1$ customer(s), then $P\left(\mathcal{T}_{a(j)} > T \mid \mathcal{T}_{a(j)} > t, N = j-1\right) = P\{Z_{N+1} > T\}$

$$= \sum_{i=1}^{j}\left(\left(\lambda_2 T\right)^i / i!\right) e^{-\lambda_2 T} \quad (3)$$

where above expression describes waiting time of j^{th} customer is j-phase Erlangian distribution and can be symbolized as $\sim \text{Erlang}(j,\lambda_2)$.

Thus, on solving further with required terms then Equation (3) is easily summarized to (see [7]), $e^{-q\lambda_2 T}$. By $e^{-q\lambda_2 T}$, it is now obvious that if Z_{N+1} is a random variable of $j+1$ customers exponential variables with mean λ_2^{-1}, and $N+1$ is a random variable with geometric distribution, independent of $\mathcal{T}_{a(1)},\ldots,\mathcal{T}_{a(j+1)}$, so the Z_{N+1} has exponential distribution with mean $\left(q\lambda_2 T\right)^{-1}$, where q is failure probability or still waiting. Thus, $\mathcal{T}_{a(j)} \sim \text{Erlang}(j,\lambda_2) = \mathcal{T}_{a(1)} \sim \exp(\lambda_1)$. If there are $j-1$ customers in the system before the j^{th}, then in order for such customer to get their service, all j must have been served at least by time T. Hence, Equations (2) and (3) too show that before system begins all customers are dependent on system commencing whereas while in queuing system except for the first customer each individual depends upon the previous customer(s) departure. But in both cases, all are distributed exponentially with different mean. Due to only one single reason, that all customers arrived before the service begins. Since the distributions of waiting times are in sequence so the comparison of individual waiting distribution before and until their departure is the same, that is $\mathcal{T}_{b(*)} \sim \exp(\lambda_1) = \mathcal{T}_{a(*)} \sim \text{Erlang}(j,\lambda_2)$.

Therefore, in an arbitrary case λ_1 parameter estimating the waiting time is distributed exponentially before the system begins and whereas λ_2 will cause the estimation of waiting time with the Erlangian distribution after the system is ready. Then, from the view point of customers the overall waiting time from arrival before the system begins until their departure is based on joint distribution of λ_1 and λ_2 then distribution of waiting of i^{th} customer before outlet begins and j^{th} customer after outlet opens, respectively, is $P\left(\mathcal{T}_{b(i)} > x, \mathcal{T}_{a(j)} > T\right)$

$= e^{-\lambda_1 x} e^{-q\lambda_2 T}$. For $\lambda_1, \lambda_2, T,\ x,\ q > 0$, so $P\left(\mathcal{T}_{b(i)} > x, \mathcal{T}_{a(j)} > T\right)$ gives joint distribution of waiting at two events and therefore $P\left(\mathcal{T}_{b(i)} > x, \mathcal{T}_{a(j)} > T\right) \approx P(Z_{N+1} > T) \approx P\left(\mathcal{T}_{b(i)} > t + x, \mathcal{T}_{b(i)} > t\right)$.

c) Simulation results during unknown opening time: The following simulation results is for various values of λ. However, the probability distribution at any time will be different. For e.g., if $\lambda_1 = 0.5$, $\lambda_2 = 0.67$ and $(\lambda_1 \cap q\lambda_2) = 0.84$, $q = 0.5$ then with their probability density function $\lambda_i e^{-\lambda_i \tau}$, $i = 1{,}2$ and time space for each λ_i is $\tau \in (0,\infty)$ respectively. When the parameter has higher value then the mean gets very small. In other words, the higher the rate of parameter the lowers the mean waiting time or the proportion for small units of time. Fig.1 illustrates the simulation results comparison during waiting time distribution before system begins and waiting time after the system begins. The waiting time before system begins is exponential distribution with parameter λ_1, as only single event occurrence is considered. Whereas, as compared to the waiting inside the system, the waiting is distributed with *j*-phase Erlangian which is higher. Since the waiting during system service becomes large because the *j* customers departure are multiple events to occur in sequence. Fig. 1 clearly shows area under the curve during *j*-phase Erlangian distribution is greater.

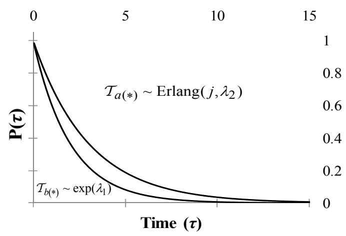

Figure 1. Waiting during unknown opening time, $\mathcal{T}_{b(*)}$ and $\mathcal{T}_{a(*)}$

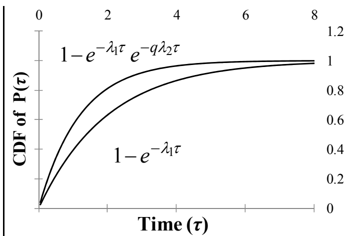

Figure 2. Cumulative distribution of $\mathcal{T}_{b(*)}$ and $\mathcal{T}_{a(*)}$

TABLE I λ_i VALUE DESCRIPTIONS $\forall \tau$

Parameter	Support	Mean	Median	Variance	Entropy
λ_1	$\tau \in (0,\infty)$	2	1.39	4	1.69
$q\lambda_2$	$\tau \in (0,\infty)$	2.99	2.07	8.91	1.40
$\lambda_1 \cap q\lambda_2$	$\tau \in (0,\infty)$	1.19	0.82	1.42	0.84

TABLE II DISTRIBUTION FUNCTION DETAILED STATISTICAL DESCRIPTION

Cumulative distribution function					
Description	*Mean*	*Std. error*	*Median*	*Std. deviation*	*variance*
λ_1	0.637	0.025	0.717	0.250	0.062
$q\lambda_2$	0.517	0.0228	0.569	0.228	0.052
$\lambda_1 \cap q\lambda_2$	0.768	0.024	0.878	0.249	0.062

Table I and Table II show the results of received data by the simulation runs for 500 counts. Table I show the mean waiting time occurring during λ_1 is small as compared to the mean waiting time occurring during λ_2 and other parameters. Statistically the customer before system commence is only dependent upon particular occurrence i.e. the beginning of server, whereas customers after opening are dependent upon many occurrences, specifically the departure of first-in customer(s). However, for infinitesimal values of time, the sum of probability density function goes to 1. The mean time occurring during joint process of before and after parameters appears smaller. This is due to the joint process been taken as overall distribution rather than individual. As the individual element changes more than that of a single set containing many elements, similarly the mean waiting time is less if the joint distribution is considered. The simulation result shown in Fig. 2 is derived by joint distribution of waiting and the exponential distribution. Fig. 2 and Table II clearly show that waiting distribution as a function of customer(s) with λ_1, while waiting accumulation appears to be slower than as compared during joint distribution with λ_2. As Table I shows the output using mathematical models described as $e^{-\lambda_1\tau}$ and $e^{-\lambda_1\tau} e^{-q\lambda_2\tau}$, where τ is common time for all scenario.

Similarly Table II shows statistical description of samples cumulative distribution from the simulation results by such equations. For exponential distribution the expected value and the standard error are analogous therefore if mean changes the other parameters will be too alter. Specifically, if customers are expected to wait a long amount of time (for example, before system begins) the amount of variability from customer to customer is expected to enlarge. If the same customers expected to wait less in time before the system begins or in the queuing system, the values show less differences from customer to customer. Hence, waiting before opening is essentially an obscure behaviour that can cause wastage during large duration of waiting times and may challenge the system arrangements after it commences.

2) Second scenario as known opening time: Under this scenario, the situation is before outlet opens and would be arrival knows outlet opening time. Therefore, to get probability of time spent by each customer before system begins is to evaluate first the average waiting time taken by customer(s). If $k \in \mathbb{N}$ customer(s) arrived at times $T_1 < \ldots < T_n$ then k_{T_1} arrived at T_1 then k_{T_2} arrived at T_2 and so on and where k is random variable at each T_i, and k can be either single or more than that. Therefore, waiting time for k_{T_i} ,before the system is activated is $k_{T_{b(i)}} = k_{T_i} + \ldots + k_{T_n}$, where k_{T_n} is customer(s) arrival time just before system starts service and $i \in \mathbb{N}$. By this then $k_{T_{b(i+1)}} = k_{T_{i+1}} + \ldots + k_{T_n}$ and so on until $k_{T_{b(n)}} = T_{b(n)}$, where $T_{b(n)} \neq 0$. The previous customers waiting times are not just because of the arrival of the $k_{T_{b(n)}}$, as $k_{T_{b(n)}} \neq 0$. It is due to system opening at fixed time and customers by their own wish arrived earlier to get fast opportunities. To evaluate time spent by early arrived customers is then based on average time spent by each customer.

Thus, on average each customer(s) spent is the median of the length of arrival times, i.e. $\dot{\mathrm{M}}\left(k_{T_{b(i)}}, \ldots, k_{T_{b(n)}}\right)$, where $\dot{\mathrm{M}}(*)$ symbolized the *'median of'*. The median is used to determine the location of reasonable time when the distribution is skewed. If $\tilde{X}$ is the average time taken before system starts service is then $\tilde{X} = \dot{\text{M}}\left(k_{T_{b(i)}},\ k_{T_{b(i+1)}},\ \ldots,\ k_{T_{b(n)}}\right)$, where $\tilde{X}$ is for those who arrived at some time. But there can be any number of k arrived together, where $(k = 1,2,3,\ldots)$. Therefore, then for each group of k arrived will spend $\tilde{X} = \dot{\text{M}}\left(k_{T_{b(i)}}, k_{T_{b(i+1)}}, \ldots, k_{T_{b(n)}}\right)k^{-1}$. The proportion of time a single customer spend while waiting for the opening of the outlet is $P_{T_{b(i)}} = \dot{\text{M}}\left(k_{T_{b(i)}}, k_{T_{b(i+1)}}, \ldots, k_{T_{b(n)}}\right)\left(k_{T_{b(n)}}\right)^{-1}$. Hence, the proportion of time a single group of customer(s) wasted their time is the median of all the times over the customer(s) who arrived just before system commence their service.

a) Simulation result during known opening time: Fig. 3 shows the collective illustration of the joint distribution of event before and event after the system begins and the cumulative of both distributions. It is interesting that the density function of overall time spent always starts at nearly 50%. Fig. 3 shows a different behaviour amongst the distribution and density function. The joint distribution cumulative function goes to one for a large number of countable times whereas area under the curve is not a valid density function until unless if $\tilde{X}_k \neq q$. Therefore, to validate the probability density of joint distribution of two events, the average waiting time before system begins must be the same as the waiting time inside the system.

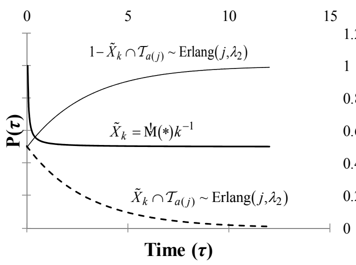

Figure 3: Waiting during known opening time: $\tilde{X}_k, \tilde{X}_k \cap T_{a(*)}$ and Cumulative waiting

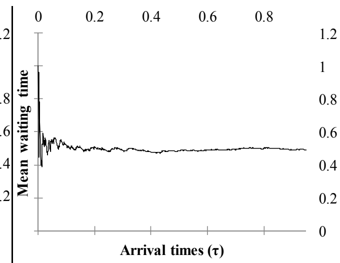

Figure 4. Early arrival times vs mean waiting time (continuous-time)

TABLE III DETAILED STATISTICAL DESCRIPTION OF $\tilde{X} \cap q\lambda_2$

Parameter	Support	Mean	Median	Variance
$\tilde{X} \cap q\lambda_2$	$\tau \in (0,\infty)$	1.5	1.03	4.45

Table III shows the joint distribution of waiting before and later until departure. The simulation results are shown in Fig. 3. In Fig. 3 the $\dot{\text{M}}(*)/k$ is the probability of $\dot{\text{M}}(*)$ over the total number of times and it behaves in similar ways to the laws of large numbers. That is, the long term probabilities remain near to 0.50 of all time averages. For distributed values on $[a,b]$, the average remains either nearly below or at nearly above to 0.50, respectively. Therefore, based on the above description say, for $T_{b(i)}$ as T_i, then $\lim_{x\to\infty, i\neq 0} P_{T_i} \approx 0.5$. That is, if time occurring at specific event is large and known and the situation when the unspecific events are occurring the overall expected time for specific event to occurs remains nearly half of the real time. In other words, if the first customer(s) reached H hours earlier and knows when the service will start, the overall waiting time until services begins is only fifty percent. This reason is not just because of opening hours of server but also the waiting time has been deduced from the times of other customer(s) arrival.

Besides that, Fig. 4 also confirms that during continuous-time, in long-run the serving system viewed the mean waiting time for all customers nears to 0.5. Thus, if the first customer proudly waited H hours than that of later arrivals, probabilistically it has only waited 0.5 of the time as well the same distribution. Independent of the would be queuing system waiting times and known to the 'exception' the customers have remained inactive in one place while expecting something but by the principles of statistics they have wasted ½ of their time. That is, for the queuing system the collective waiting time for the all customers is unprejudiced. This situation can leads to either non-queue state or balking. Thus, this is also an 'exception' that can weaken the serving system reputation.

B. *Waiting Time Distribution During Brazenness*

The various 'exceptions' that challenges the patience of the customers has been mentioned in subsections *A.1)*, *A.3)*, *C.1)* and *C.2)*. This 'exception' largely happens where the conservatism practices are higher. Rather than the server preempts the customer for service, the current customer is shifted by an inconsideration act. The server has assumed not to involve in any sort of customers engagement. Hence, the service system is state independent and identically distributed. The displaced act may reposition the affected customers either by single or more position back while in queue. In single server queuing system with Poisson arrivals with parameter λ and inter-arrival times and service times are exponentially distributed with mean rate $1/\lambda$ and $1/\mu$, respectively. The customers are served on a first-come first-serve basis and all customers require waiting in that is essential for the service. The displaced customer reposition back by any counts is the same as the customer who just arrived and finds many customers already waiting in the queue.

Due to the 'exception', the changes in the behaviour of customer arrival occur inside the system when there is already a queue. Therefore, the customer who got displaced while already in the queue is affected. Furthermore, for the affected customer caused by an 'exception', λ of customer(s) is not in concern and the waiting time will depend upon the number of times it has been displaced back. As a result, the distribution for the waiting time of the displaced customer shall be evaluated directly without the concern of arrival rate. The 'exception' that causes displacement is assumed to occur while a customer is at the head of the queue and about to enter service. If W_s is the random variable that identify the waiting time in the system then $F_{Z_N}(T)$ is the cumulative waiting time in the queuing system, where Z_N is sum of random variables including displaced customer. Therefore, if the i^{th} customer is at head of the queue and has been displaced by $w-1 \geq 0$ additional customers, then, X_1 is the elapsed time from $t = 0$ till the customer(s) (that source 'exception') at head of the queue enter the service, then X_i, $(i = 2,3,\ldots,w)$ is the time length that i^{th} customer used up in waiting to get out of the system. That is, for i^{th} customer the service time length is X_w, then obviously the X_i^{th} total time to depart from the system is $X_1 + X_2 + ... + X_w$, where X_i^{th} is same as w^{th} customer or affected customer.

It states that the affected customer has to wait through w exponential service times including their own. For the consideration of waiting time in system during an 'exception', then the chances that the system is empty or no customer in queue is not currently applicable here. Therefore, the direct calculation is applied by considering only service rate regardless of the arrival rate. If Z_N is the random variable for the sum of $X_1 + \ ... + X_w$ are independent, identically distributed and exponential variable, where $w \in \mathbb{Z}_+$ discrete random variable. If the geometric distribution is considered for $N-i$ random variables then conditional probability that $N-i \ = m$, given that $N \geq i$, $(i = 0,1,2,...)$ as shown as $P(N = i + m \mid N \geq i) = P(N = i + m, N \geq i)/P(N \geq i) \ = q^m p$. If we assume, for $N-1$ failures the geometric distribution is $P(N = m+1) \ = q^m p$, $(m = 0,1,...)$, then, the waiting distribution probability for the affected customer is the multiplication of probability of the number of customers ahead and the probability of waiting for that customer is $\sum_{w=1}^{\infty} P(Z_N > T \mid N = w) P(N = w)$. Thus, from Equation (3) and with law of total probability, that $P(Z_N > T)$ is

$$P(Z_N > T) = \sum_{m=0}^{\infty} \sum_{n=0}^{m} \left((\mu T)^n / n! \right) e^{-\mu T} q^m p \qquad (4)$$

If the number of customer(s) cause 'exception' is considered to be finite then the waiting should be finite. Therefore, by substitution the arrangement of summation of above expression, the waiting in the system occurring at most w times in the interval $(0, T]$. Therefore, then $F_{Z_N}(T)$

$$P(W_s \leq T) = 1 - \sum_{n=0}^{w} \sum_{m=n}^{w} \left((\mu T)^n / n! \right) e^{-\mu T} q^m p \qquad (5)$$

where $F_{Z_N}(T)$ is the cumulative distribution for Z_N. If $X_1, X_2, ...$ are mutually independent random variables each with an exponential distribution with parameter μ, then their sum Z has an incomplete gamma distribution with parameter w and μ then by expanding the only term $\sum_{n=0}^{w} \left((\mu T)^n / n! \right)$ $e^{-\mu T}$, it gives $\Gamma_w(T) = e^{-\mu T} \ \{ \Gamma(w+1 \ , \mu T) / \Gamma(w \ +1) \}$, $T \geq 0$.

But if together with geometric distribution in Equation (5) second term then it is characterize as each n variable distribution convolution with $n-i$ distribution of m variables distribution, where $(i = 0,1,2,...)$, respectively for the each sequence of n. Therefore, by arbitrary substitution the reduce form of Equation (5) for a finite distribution and on further expansion with $q = (1-p)$, then the summation series gives

$$= 1 - e^{-(\mu T)} \sum_{n=0}^{w} \left(\left((\mu T)^n / n! \right) \left((1-p)^n - (1-p)^{w+1} \right) \right) \qquad (6)$$

By solving further Equation (6) and then their compilation, and then $F_{Z_N}(T)$ is

$$= 1 - e^{-\mu T} \left(\left(e^{-\mu p T} \Gamma(w+1, \mu T q) - q^w \Gamma(w+1, \mu T) \right) / \Gamma(w) \right) \qquad (7)$$

Therefore, Equation (7) presents the distribution function of X_i^{th} which got $w-1$ displaced that $P(W_s \leq T)$. For $P(Z_N > T)$ the distribution is simply the complement of Equation (7).

1) Simulation results: The distribution for waiting time by Equation (7) is given in Fig. 5, where $P(Z_N > T)$ is infinitely small, the probability of waiting X_i^{th} displaced customer is large. However, as time becomes large, the waiting probability also decreases because by the time the unethical customer(s) shall be served too. The server has no problem in serving as long as there is first-come first-serve unbroken queue discipline. Though, not only the X_i^{th} customer is affected but the entire later customers from X_i^{th} are affected too. Hence, if the X_i^{th} customer gets to wait more than before being displaced then there are chances that the queue may be broken and may leads to the non-queue state. The waiting of displaced customer, X_i^{th} can also described as w-phase Erlangian distribution. In which the X_i^{th} get to wait $w-1$ number of 'exception' excluding their own time in the queue.

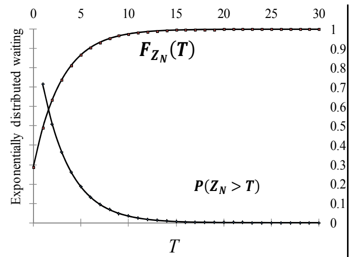

Figure 5: CDF Waiting by Markovian service

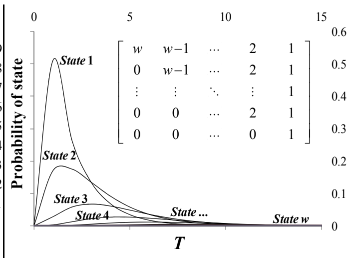

Figure 6: Erlangian waiting distribution function for different phases (states)

Displacement induced by an 'exception' can also extend through the view of discrete-time Markov chain. The interesting part is that the Markov chain transition matrix form is similar to *w*-phase. If the different phases can represent as the state space elements then being in some state can represent waiting while α is ahead. Such view given in the form of transition matrix is not for any solution, it is just to relax the understanding procedure of waiting. Therefore, in the transition matrix of such form (inserted in Fig. 6), if the column side is taken as state I then the row is state J, where $(I = 1,2,...,w)$ and $(J = w, w-1,...,1)$, respectively. The transition of chain from state 1 to state w will select all the waiting time of w customers including the affected one during state 1 i.e. the probability of waiting customer at w^{th} is very large. Therefore, the jump discontinuity in Fig. 5 at $T = 0$ for brazenness occur due fact that there is positive probability that X_i^{th} customer is displaced by interposed customers.

The waiting distribution decrease as $T \to \infty$. If the chain is at state 2 the waiting time for $w-1$ customers noted for different estimated times. In this state the waiting probability is less than as compared with when w customers waiting time in process. Thus, as the state goes to w the waiting time probability goes to zero. Apart from all in general, if the chain in the above matrix is converging to departure but the opposite of chain is the divergence of departure. Thus, the divergence of departure is basically the blocking of departures.

Fig. 6 is derived by using the Equation (7) and provides sample similar to the above discussion. The probabilities are drawn for different values of T for the w^{th} customer while in state 1. As the chain makes transition to another state, meaning that the customer(s) caused by 'exception' are departing. When the space elements become less, then the probability of waiting more for various times also decreases and so on. However, the evaluation is done to notice waiting time if the affected customer has no problem in waiting for a long period. But practically the affected customers have to wait too long if compared unconditionally without 'exception'. Then, this 'exception' surely challenges the tolerance of customers that caused the queue breaking.

C. Waiting Time Distribution During Service Variations

The possible changes in server conduct normally can occur while there is already a queue in the system. The arrival of customer has been served on first-come first-serve basis. The 'exception' that disrupts services may cause the server to perform at distinct service rates. The service due to 'exception' may experience a change in their services rate. When the 'exception' occurs there are chances that the mean service time of server increases. If this is so, then the queuing system will encounter congestion. If we considered a case in which a server is serving the customers with service rate μ_a and provides no limitation on arrivals. If the unpredicted 'exception' occurs, then $\mu_a > \lambda$ and when a 'exception' occurs then the mean service time shall be different from μ_a. If suppose the 'exception' occurs at near to i^{th} customer then the change in service rate μ_b later than some i^{th} customer, where $\mu_a > \mu_b$. For a balanced system $\mu_b > \lambda$. For this section $\mu_a > \lambda$ and $\mu_b > \mu_a$ which is same as $\mu_b < \lambda$. Analogous to [2], if μ_m is the combined rate of μ_a and μ_b, then 'exception' occurs the mean rate μ_m becomes depends upon the state of the queue. Therefore, by considering the service time as Markovian μ_m is given as $\mu_m = \{\mu_a,(1 \le i < m);\mu_b,(i \ge m)\}$.

If the arrival rate is based on Poisson process with constant parameter λ and the probability of the number of customers in the system for M/M/1 queuing system as $p_m = \{p_0(\lambda_{m-1}\cdots \ldots\lambda_0/\mu_m\ldots\mu_1);p_0\prod_{i=1}^{m}\lambda_{i-1}\mu_i^{-1} = p_0(\lambda/\mu)^m\}$. If $\lambda\mu^{-1} = \rho$, then $p_m = (1-\rho)\rho^m = p_0\sum_{m=0}^{\infty}\rho^m = p_0\rho^m$ and for distinct traffic intensity $p_m = (\rho_0\rho_2\ldots\rho_m)p_o$. If there are only two types of traffic intensity sets for ρ_a there are $i-1$ quantities and for ρ_b there are $m-i+1$, and then followed by

$$p_m = \rho_a^{i-1}\rho_b^{m-i+1}p_o \tag{8}$$

where $\rho_a = \lambda\mu_a^{-1} < 1$ and $\rho_b = \lambda\mu_b^{-1}$. Also it can be assumed that $\rho_b \ge 1$. Similarly, for ρ_a if there are m quantities then $p_m = \rho_a^m p_o$. For p_0, if assume that the sum of $\{p_m\}$ must equals to 1, so $p_0 = \left(\sum_{m=0}^{\infty}\rho^m\right)^{-1}$.

Consider the quantities from $(0 \to \infty)$ is divided into $(0 \to i-1)$ and $(i \to \infty)$. In addition, the ρ^m split up into ρ_a^m and $\rho_a^{i-1}\rho_b^{m-i+1}$. As the series is $\rho^m = (1+\rho_a+\rho_a^2+\ldots+\rho_a^{i-1})+\left(\rho_b^1+\rho_b^2+\ldots+\rho^{\infty}\right)$, Therefore, by combining all the terms then it gives in $\rho^m = \left(\sum_{m=0}^{i-1}\rho_a^m+\sum_{m=i}^{\infty}\rho_a^{i-1}\rho_b^{m-i+1}\right)$ by putting such expression in $p_0 = \left(\sum_{m=0}^{\infty}\rho^m\right)^{-1}$ and it provides

$$p_0 = \left(\sum_{m=0}^{i-1}\rho_a^m+\sum_{m=i}^{\infty}\rho_a^{i-1}\rho_b^{m-i+1}\right)^{-1} \tag{9}$$

which is simply reduce to

$$p_0 = (1-\rho_b)(\rho_a-1)/\rho_a{}^i - \rho_b\rho_a{}^{i-1} + \rho_b - 1 \tag{10}$$

for $(\rho_a < 1, \rho_b < 1)$. Since the section is assuming two types of traffic intensity in which one of the traffic intensity considered as greater than or equals to 1. Therefore, if $\rho_b = 1$ then Equation (9) becomes

$$p_0 = (1-\rho_a)\rho_a/\rho_a + \rho_a^i - 2\rho_a^{i+1} \tag{11}$$

Equations (10) and (11) are the two results of p_0 to get the expected size of the queuing system. If N is the number of customers in system in steady state and L represent its expected value the $L = \mathrm{E}[N] = \sum_{m=0}^{\infty}mp_m$ and as $p_m = (1-\rho)\rho^m$ then $= p_0\sum_{m=0}^{\infty}m\rho^m = mp_0\left(\sum_{m=0}^{i-1}\rho_a^m+\sum_{m=i}^{\infty}\rho_a^{i-1}\rho_b^{m-i+1}\right)$. On solving further, then expected size, L is

$$= p_0\left(\frac{i\rho_a^{i+1}-\rho_a^{i+1}-i\rho_a^i+\rho_a}{(\rho_a-1)^2} - \frac{\rho_b\rho_a^{i-1}(i\rho_b-\rho_b-i)}{(\rho_b-1)^2}\right) \tag{12}$$

as $L = \mathrm{E}[\text{customers in queue}] - \rho$ i.e. $L = L_q - \rho = L_q - (1-\rho_0)$. Thence, by Little formula the waiting time in the system is $W_s = L\lambda^{-1}$, where W_s is the waiting time in the system with constant rate of arrival. So $W_q = (L-1+p_0/\lambda) = L_q\lambda^{-1}$, where W_q is the waiting time in queue.

1) Simulation results: Fig. 7 shows the simulated results of waiting time in the system as well in queue. If $\mu_a = \mu_b$, then the waiting times during both rates are same. That is, for each customer waiting inside the system shall be same. For 'exception' occurring near to some i^{th} customer, it follows that $\mu_b < \mu_a$, μ_b is assumed to be greater than 1, then from Equation (11) the value of p_0 is evaluated for each sequence of customers number in system. When the value of p_0 is put in Equation (12) then the expected number of customer in the system is evaluated.

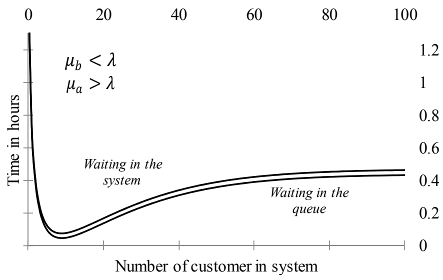

Figure 7: Waiting inside the queuing system during ‘exception’

The behaviour of Fig. 7 is approximating similar to Fig. 3 where probability of median has been evaluated. Due to ‘exception’ that caused the changes in the service rate and it further caused fluctuation in the expected number of customers in system. When the mean service time changes or increases the number of customer begin to increase.

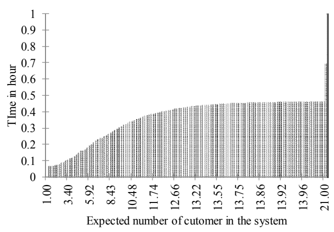

Figure 8: W_s of L

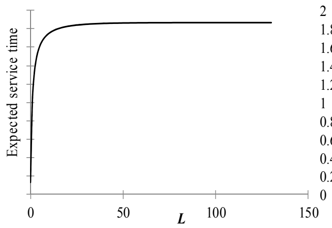

Figure 9: Expected service time for each customer

With constant arrival and decreasing in the service the waiting time for the expected number of customers in the system are increasing as shown in Fig. 8. When the service rate decreases the queue begins to become large and the system surely going to be overcrowded. As shown in Fig. 7 due to fluctuation in service rate $W_q \approx W_s$ and for each customer (if) once their service comes they will be served immediate but it requires large waiting time. Also, if μ_b is same for later customers then for the long term the waiting time in queue as well in waiting time in system will achieve a stationary value. Even if the value becomes stationary, this does not reduce the number of customers remaining in the system. With the μ_b service rate and the same waiting time for each then there shall be same number of customer remaining each time in the system and this means the system will never be empty. Since, the μ_m is state dependent and by W_s and W_q, then the mean service time for each customer in system will be $(1-p_0)\lambda^{-1}$. Therefore, the mean service time for each customer shown in Fig. 9 which shows that for initial customer the service time is varying. In Fig. 9 as μ_m^{-1} is changing function of increasing L, after a number of customers the service rate rapidly changes. Even during μ_b is not varying but this will not empty the serving system In Thus, if changes of system service rates results in higher waiting time and large number of customers will remain in the system. Subsequently, the system has higher chances of approaching to non-queue state via overcrowding.

IV. Conclusion

This paper had discussed various ‘exceptions’. The waiting probability models simulation showed that during various ‘exception’ the queuing system gets and drawn large amount of times to departure. During early arrival and brazenness affects on partial customers ‘exception’ cause the waiting probability to cumulate faster. The service variations ‘exception’ similar to early arrival, the joint waiting would be then waiting before service variation then waiting until departure. Besides as compare to brazenness, there is interposed of changes in service rate. In fact, the large waiting brought in by induced of server delay, induced of customers and induced of different service rate. The sequential affects of ‘exception’ commonly brings down the system into non-queue state. The term non-queue state is a serving system without formal queue at all or arbitrary positions of the customers. The ‘exception’ has whole chances to overcome the queuing process. The non-queue state is basically the affected queuing system when $\rho \geq 1$. The continuous ignorance of ‘exception’ generates non-queue state. The paper signifies that study of ‘exception’ can support to understand the reasons of delays in services, which can cause system efficiency problems. Moreover, if the influence of ‘exception’ has not been figured out then there will be no service during non-queue state, and this will have an auxiliary economical issues. Thus, the factor of ‘exception’ must be involved in the study of queuing system to model reality. The non-queue will be discussed in the next article under process.

References

[1] A. Kokkinou, and D. A. Cranage, “Modeling human behavior in customer-based processes: The use of scenario-based surveys”. In: S. Jain, R.R. Creasey, J. Himmelspach, K.P. White, and M. Fu, eds., Proceedings of the 2011 Winter Simulation Conference. IEEE, 2011.

[2] D. Gross, J. F. Shortle, J. M. Thompson and C. M. Harris,. Fundamentals of queueing theory, 4th ed., New Jersy: Wiley Publishing, Inc., 2008, pp.1-53

[3] D. Yue, W. Yue, and G. Xu,. “Analysis of a queueing System with impatient customers and working vacations”. Proceedings of the 6th International Conference on Queueing Theory and Network Applications, New York, USA, 2011, pp.208-212.

[4] J. W. Cohen, The single server queue, Revise ed., Amsterdam: North-Holland Publishing Company, 1982, pp. 1-20.

[5] K. Wang, Na Li and Z. Jiang, “Queueing system with impatient customers: A review”. IEEE, 2010.

[6] L. Kleinrock, Queueing Systems, Vol. 1. Wiley-Interscience Publication, New York, 1975, pp. 3-53.

[7] R. B. Cooper,. Introduction to queueing theory, 2nd ed., London: Edward Arnold, 1981, pp. 9-71.

[8] T. R. Robbins, D. J. Medeiros and P. Dum, “Evaluating arrivals rate uncertainty in call centers”. In: L.F. Perrone, F.P. Wieland, J. Liu, B.G. Lawson, D.M. Nicol, and R.M. Fujimoto, eds., Proceedings of the 2006 Winter Simulation Conference. IEEE.

[9] U. N. Bhat, An Introduction to Queueing Theory Modeling and Analysis in Applications, Boston: Birkhäuser, 2008, pp. 3-24.

[10] U. Yechiali, Queues with system disasters and impatient customers when system is down. Queueing system, vol. 56. 2007, pp. 195-202.