International Journal of Computer and Information Technology (ISSN: 2279 – 0764)
Volume 04 – Issue 02, March 2015

Combined MMSE-OSIC MIMO Detector using Detection Error Estimation

Ilia Georgiev Iliev
Department of Radio Communications and Video Technologies,
Technical University of Sofia, Bulgaria
Email: igiliev [AT] tu-sofia.bg

Abstract— **This paper proposes an algorithm for detection in multiple input multiple output (MIMO) communication systems. The algorithm combines minimum mean square error (MMSE) and minimum mean square error with ordered successive interference cancellation (MMSE-OSIC) detectors with detection error estimation. Neyman-Pearson criterion is used and the decision boundary is derived. It helps the implementation of the proposed algorithm in practical systems and the achieving the balance between the computational complexity and the bit error rate performance for different signal-to-noise ratios. The derived adaptive detection boundary can be applied in various other combined detector schemes. The analytical and simulation results show that the average computational complexity of the proposed algorithm approaches to the MMSE equalizer, and the error probability is comparable with the conventional MMSE-OSIC detector.**

Keywords— Equalization, Detection, Neyman-Pearson criteria, Minimum-mean-squared error with ordered successive interference cancellation (MMSE-OSIC), Multiple Input Multiple Output (MIMO).

I. INTRODUCTION

There exist multiple publications and practical applications of different algorithms for MIMO forming and detection. All of them count on for two major characteristics: computational complexity and bit error rate (BER) performance. The algorithms using detection with minimum mean square error (MMSE) or ordered successive interference cancellation (OSIC), have a reduced computational complexity and worse noise immunity compared to the optimal maximum likelihood (ML) detector [1-4]. Due to the matrix orthogonalization of the MIMO channel, the set of algorithms with lattice-reduction (LR) [5,6,8] and subsequent suboptimal detection tends to the error performance of ML detector. There exist techniques that use combination of linear equalization and subsequent detection with ML detector [7].

In this work the idea from [7] is elaborated and a modification of the MMSE-OSIC detector in order to reduce the average computational complexity and remaining the noise immunity according to the original case is proposed. Here is applied Neyman-Pearson criterion for detection error estimation and the derived adaptive detection boundary helps achieving a balance between the computational complexity and error probability for different signal-to-noise ratios (SNR). The derived adaptive detection boundary and the rule for detection error estimation can be applied in various combined detector schemes. In this work a new combined method is investigated to prove analytical results. The method combines MMSE and MMSE-OSIC algorithms using an estimation of the detection error.

The performance of the conventional MIMO detectors with MMSE, MMSE-OSIC, ML processing and the proposed modified method CMMSE-OSIC is investigated and compared each other. The relative number of calculations by the second detector is much smaller than that obtained in [7], because the derived dependencies allow for adaptively changing the decision boundary

Notation: The operator $[]^T$ denotes matrix transpose, and $[]^H$- Hermitian transpose. $|x|$ is absolute value or a module of a complex scalar number, $||\mathbf{x}||$ denotes Euclidian norm of a vector $\mathbf{x}$, $\mathbf{I}_{N_t}$ is unity matrix with size $N_t \times N_t$. $N(\mu_n, \sigma_n^2)$ is Gaussian distribution with mean value μ_n and variance σ^2_n.

II. SYSTEM MODEL

Mathematical description of the MIMO system in base band versus the received vector $\mathbf{r}$, is given by the matrix equation:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} . \quad (1)$$

The input data of the transmitter is demultiplexed in N_t layers, where they are modulated with M-QAM modulation and transmitted in parallel in N_t channels, which can be space, time or frequency diverted. Each symbol s_m of the input column vector $\mathbf{s}$ is presented in the baseband. $s_m \in A$ and A is a finite set, consisting of the complex elements of the constellation of the M-QAM modulated signal. The average power of the modulated signals in each channel is normalized and $E\{\mathbf{s}\,\mathbf{s}^H\} = \mathbf{I}_{N_t}$.

The receiver has N_r parallel input channels, which are affected by additive white Gaussian noise (AWGN) n_k. A noise vector $\mathbf{n}$ is defined as complex elements with Gaussian distribution with zero mean value and variance σ^2_n, $E\{\mathbf{n}\mathbf{n}^T\} = \sigma^2_n.\mathbf{I}_{N_r}$. The elements $h_{k,m}$ of matrix $\mathbf{H}$ with size $N_t \times N_r$ are with random complex values and model the transmission coefficient between the m-th layer of transmitter and k-th receiver. Their statistical properties depend on the type of the fading in the channel. The transmission coefficients are prior known in the receiver and they are independent complex random values with normalized variance equal to unity. The fading type is flat slow Rayleigh fading. The channel is invariant, when a single block **s** is transmitted. The real and imaginary parts of $h_{k,m}$ are Gaussian distributed - $h_{R_{k.m}}, h_{I_{k.m}} \sim N(0{,}0.5)$.

III. PROPOSED ALGORITHM

The algorithm can be described in the following way:

1. The received signal **r** is detected with MMSE detector and $\hat{\mathbf{s}}_{MMSE}$ is the detected vector;
2. Calculate the estimation of the detection error in a decision taken based on the metric criterion for ML:
$$\varepsilon = \left\|\mathbf{r} - \mathbf{H}\,\hat{\mathbf{s}}_{MMSE}\right\|^2 / \sigma_n^2 = \left\|\mathbf{H}(\mathbf{s} - \hat{\mathbf{s}}_{MMSE}) + \mathbf{n}\right\|^2 / \sigma_n^2 \,; \qquad (2)$$
3. Detection of the error. If the assessment in not bigger than predefined limit L_ε ($\varepsilon < L_\varepsilon$), the processing is stopped and the output result is $\hat{\mathbf{s}}_{MMSE}$;
4. If $\varepsilon \geq L_\varepsilon$, additional detection with MMSE-OSIC is applied. It has higher computational complexity and higher noise immunity.

The assessment ε is the normalized Euclidian distance in the space of the received signal vectors and it is used in [7] in order to define the “reliability judge rule”.

IV. STATISTICAL PROPERTIES OF THE DETECTION ERROR ESTIMATION

After detection with MMSE the number of error symbols is N_e. The estimation ε can be defined by (1) and (2) with:

$$\varepsilon = \frac{1}{\sigma_n^2} \sum_{k=1}^{N_r} \left| \sum_{m=1}^{N_e} h_{k,m}.\Delta s_m + n_k \right|^2 = \sum_{k=1}^{N_r} \left|\varepsilon_k\right|^2 . \qquad (3)$$

In baseband, the difference between the transmitted and the received vector from the m-th layer is Δs_m and $h_{k,m}$, n_k are complex values. Consequently:

$$\varepsilon = \sum_{k=1}^{N_r} \mathrm{Re}\{\varepsilon_k\}^2 + \sum_{k=1}^{N_r} \mathrm{Im}\{\varepsilon_k\}^2 . \qquad (4)$$

$$\mathrm{Re}\{\varepsilon_k\} = \left(\sum_{m=1}^{N_e} h_{R_{k,m}} \Delta s_{R_m} - \sum_{m=1}^{N_e} h_{I_{k,m}} \Delta s_{I_m} + n_{R_k} \right) / \sigma_n \,,$$

$$\mathrm{Im}\{\varepsilon_k\} = \left(\sum_{m=1}^{N_e} h_{R_{k,m}} \Delta s_{I_m} + \sum_{m=1}^{N_e} h_{I_{k,m}} \Delta s_{R_m} + n_{I_k} \right) / \sigma_n \,.$$

n_{R_k} and n_{I_k} are the components of the complex AWGN or $n_{R_k}, n_{I_k} \sim N(0, \sigma_n^2/2)$.

The worst case, in which the decision was taken with ε is the vector in which the error between the transmitted and the received vector Δs_m has the smallest possible amplitude d_{min} for all transmitting channels. The simplification is caused by the fact that the most likely errors are between adjacent vectors. If it is used M-QAM and the error is only between two adjacent characters of the vector constellation, Δs_m is either pure real or pure imaginary value, equal to d_{min}. Therefore, Re$\{\varepsilon_k\}$ or Im$\{\varepsilon_k\}$ are random values with a Gaussian distribution with zero mean value and variance:

$$\sigma_x^2 = 0.5 N_e\, d_{\min}^2 / \sigma_n^2 + 0.5 \,. \qquad (5)$$

Representing d_{min} in M array QAM constellation diagram with the signal-to-noise ratio $SNR = P_s / P_n = E_s / N_o$, the variance is:

$$\sigma_x^2 = 3N_e\, SNR / [2(M-1)] + 0.5 \,, \qquad (6)$$

where P_s and E_s are the average power and energy of the modulated signal, and P_n and N_o are the power and the noise power spectral density.

It is clear that the random variable $\varepsilon' = \varepsilon / \sigma_x^2$ has chi-squared distribution with $2N_r$ degrees of freedom. Because of $\sigma_x^2 > 0$ and σ_x^2 is independent of N_r, then $\varepsilon = \varepsilon' \sigma_x^2$ is approximated with Gamma distribution with shape parameter N_r and scale $2\sigma_x^2$:

$$f(\varepsilon; N_r, 2\sigma_x^2) = \left[\varepsilon^{N_r - 1} \exp\left(-\frac{\varepsilon}{2\sigma_x^2} \right) \right] / \left[2^{N_r} \sigma_x^{2N_r} \Gamma(N_r) \right] . \quad (7)$$

Here Γ( ) is the gamma function. When $N_e = 0$ and $\sigma_x^2 = 0.5$:

$$f(\varepsilon; N_r, 1) = \frac{\varepsilon^{N_r - 1}}{\exp(\varepsilon)\Gamma(N_r)} . \qquad (8)$$

V. DETECTION ERROR AND DECISION BOUNDARY

Let there are two random events: m_0 is the event in which there is no error after detection of the MMSE detector, and m_1 is the event when present at least one error, and in that case must apply detection with higher accuracy. In order to simplify the problem it is assumed that a priori probabilities for these random events are unknown. The Neyman-Pearson criterion is suitable for the detection error and taking a decision for the next detection. The decision for the event m_1 is based on the inequality of likelihood:

$$f(\varepsilon; N_r, 2\sigma_x^2 \mid m_1) \big/ f(\varepsilon; N_r, 1 \mid m_0) \geq \eta \,. \qquad (9)$$

The inequality (9) has an analytical solution and allows defining the decision boundary for a given threshold η and SNR:

$$\begin{aligned} \varepsilon \geq L_\varepsilon(\eta, N_e, SNR) = \\ = \left(1 + \frac{M-1}{3N_e\, SNR} \right) \ln\left[\eta \left(1 + 3N_e \frac{SNR}{M-1} \right)^{N_R} \right] . \end{aligned} \qquad (10)$$

The event “false alarm” is associated with the case, when there are no errors after detection from the first algorithm, but the decision from (10) is false - the detection is done by the second algorithm. The probability for false alarm is:

$$p_{FA}(\varepsilon > L_\varepsilon \mid m_0) = \int_{L_\varepsilon}^{\infty} f(\varepsilon; N_r, 1 \mid m_0)\, d\varepsilon \,. \qquad (11)$$

The correct decision is the event, where the estimation shows that there is a detection error from the first algorithm and it is taken the right decision for subsequent detection with the second algorithm. Probability of correct decision is determined by:

$$p_D(\varepsilon > L_\varepsilon \mid m_1) = \int_{L_\varepsilon}^{\infty} f(\varepsilon; N_r, 2\sigma_x^2 \mid m_1)\, d\varepsilon \,. \qquad (12)$$

The relation between p_D and p_{FA} is with ε and describes the response of the detector error. Such a response is shown on Fig.1 in the case L_ε is obtained for N_e=1, QPSK and MIMO $N_t = N_r = 4$. As the probability p_{FA} is smaller, the less will be applied the second detection algorithm and the average computational complexity of the combined detection will be close to that of the first algorithm. From Fig. 1, it is seen that in this case it reduces the probability of a correct decision p_D, which means that it will reduce the accuracy of the detection of the combined algorithm. For large signal-to-noise ratios detection error accuracy is high and the average computational complexity of the combined algorithm is close to that of the first detector.

Based on (10), (11) and (12), it can be determined the detector responses for different M-array modulation schematics and different number of channels in the transmitter and receiver. The threshold η is chosen by the detector response (Fig. 1), making a tradeoff between computational complexity and accuracy of detection algorithm.

The decision boundary L_ε is calculated by (10) and it appears that it changes with the change in *SNR*. *SNR* is known at the receiver and it is necessary to implement the MMSE filtering. It is recommended the parameter N_e is N_e = 1, otherwise the probability of correct detection of a single error will be small.

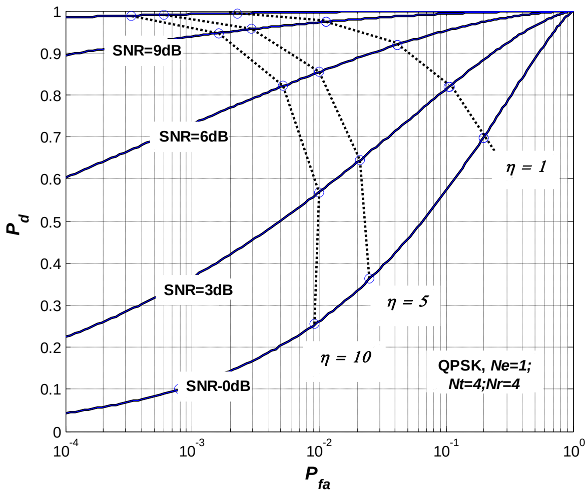

Figure 1. Error detector response

VI. COMPLEXITY AND PERFORMANCE EVOLUTION

The characteristics of the proposed algorithm are studied using a programming model in baseband in perfect synchronization between the transmitter and receiver. The parameters of the communication channel are the same as described in Section II. Fig. 2 shows the results of the BER performance depending on the ratio of the energy for transmission of one bit to the noise energy E_b/N_o.

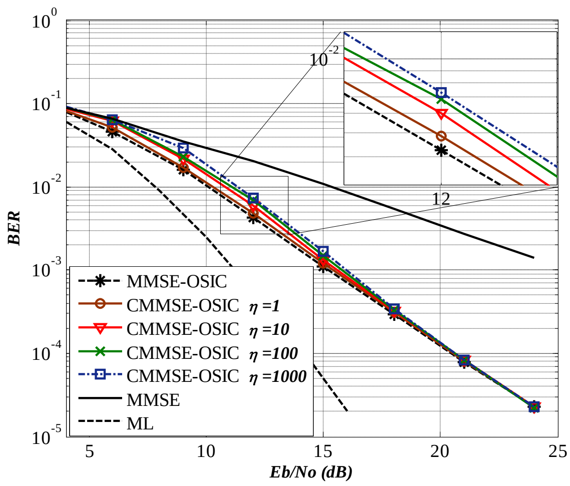

Figure 2. BER performance comparisons of the proposed CMMSE-OSIC detector with the classical MMSE, MMSE-OSIC and ML detectors when decision threshold η = 1,10,100,1000.

It can be compared the characteristics of conventional ML, MMSE, MMSE-OSIC detectors with the proposed CMMSE-OSIC method for a different decision threshold η = 1,10,100,1000. The characteristics at different parameter η can be evaluated with the same figure in the zoomed window. Fig. 3 shows graphs of the change in the relative number of references to the second detector MMSE-OSIC to as a function of signal to noise ratio. The results for both figures refer to the QPSK modulation and the number of MIMO channels $N_t = 4$ and $N_r = 4$.

The results in Fig. 2 show that for the threshold $\eta = 1$, the noise immunity of the proposed method almost coincides with that of the secondary detector with higher computational complexity MMSE-OSIC. For all decision thresholds, it is observed that at low signal to noise ratio, the noise immunity tends to primary detector MMSE. The trend increases with increasing the decision threshold, but at the expense of reducing the average computational complexity, which is evident from Fig. 3.

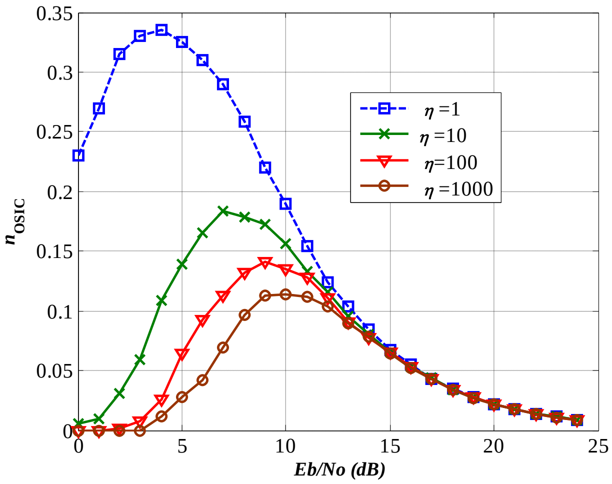

Figure 3. Relative number MMSE-OSIC calls versus E_b/N_o.

For large *SNR* and for all tested thresholds, the differences between noise immunity of the proposed method and conventional MMSE-OSIC detector are negligible. The loss of BER performance at E_b/N_o = 12dB has a maximum at η = 1000 and it is equal to 1.2dB.

The average computational complexity can be estimated by the results shown on Fig. 3. The relative number n_{OSIC} is a statistical evaluation and it is defined as the ratio of the number of requests to the secondary MMSE-OSIC detector to the total number of combined detector. If O_{MMSE} is the computational complexity of the first detector and $O_{MMSE\text{-}OSIC}$ is the second, the computational complexity of the combined algorithm is O_C:

$$O_C = O_{MMSE} + n_{OSIC}(O_{MMSE-OSIC} - O_{MMSE}). \qquad (13)$$

From (13) and from the presented figures it can be seen that the average computational complexity of the combined process is much smaller than that of the conventional MMSE-OSIC, while at high *SNR* it is close to that of the primary coarse MMSE detector. The noise immunity is comparable to that of the secondary more accurate MMSE-OSIC detector. It should be noted that the actual computational complexity of the combined method is smaller than that of (13), because the MMSE-OSIC detector uses a filter matrix that has already been determined by the first detector. This is the reason to propose a combination of these two detection methods. The relative number n_{OSIC} requests to the second detector are much smaller than that obtained in [7], because the derived inequality (10) allows adaptive change the decision boundary. The results validate the analytical formulas in Section V from the detection characteristic in Fig.1. The choice of the decision boundary controls not only the probability of false alarm and correct solution, but the computational complexity and noise immunity.

VII. Conclusion

This paper proposes a new detector in MIMO systems, which is a combination of MMSE and MMSE-OSIC algorithms through an estimation of the detection error. The obtained dependencies and the results allow, in practical applications, a compromise to determine the decision boundary, and to obtain the average computational complexity, which is close to that of the MMSE equalizer and noise immunity is comparable to the conventional MMSE-OSIC detector. The adaptive change of the derived decision boundary depends on SNR and allows minimizing the average computational complexity for satisfactory BER performance and can be applied in various other combined detector schemes.

References

[1] A. Chan, and I. Lee, “A New Reduced-Complexity Sphere Decoder For Multiple Antenna Systems,” *IEEE Conf. Commun.*, vol. 1, pp.460-464, May 2002.

[2] M. O. Damen, H. E. Gamal, and G. Caire, “On Maximum-Likelihood Detection and the Search for the Closest Lattice Point,” *IEEE Trans.on Info. Th.*, vol. 49, no. 10, Oct. 2003.

[3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," The Bell Sys. Tech. Journal, 1996, No. 1, pp. 41-59.

[4] P.W. Wolniansky, G. J. Foschini, G. D. Golden, R. A. Valenzuela, "V-BLAST: An Architecture for realizing very high data rates over the rich scattering wireless channel," *1998 URSI International Symposium on Signals, Systems, and Electronics*, , pp. 295-300, 29 Sep.-2 Oct. 1998.

[5] D. Wübben, R. Böhnke, V. Kühn, and K. Kammeyer, “Nearmaximum-likelihood detection of MIMO systems using MMSE based lattice-reduction,” *IEEE Conf. on Commun.*, vol. 2, pp. 798-802, June. 2004.

[6] D. Wübben, R. Böhnke, V. Kühn, and K. Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition",. in *IEEE Proc. VTC-Fall*, Orlando, Florida, USA, October 2003.

[7] F. Wang,Y. Xiong, X. Yang. Approximate ML Detection Based on MMSE for MIMO Systems, *PIERS ONLINE*, VOL. 3, No. 4, pp.475-480, 2007,. Available: http://piers.org/piersonline/

[8] J Pan, WK Ma, J Jalden, MIMO Detection by Lagrangian Dual Maximum-Likelihood Relaxation: Reinterpreting Regularized Lattice Decoding, *IEEE Transactions on Signal Processing*, VOL.62 No. 2, pp.511, 2014