

Robust MIMO Radar Waveform Optimization to Improve the Worst-Case Detection Performance

Hongyan Wang

College of Information Engineering
Dalian University
Dalian, China
Email: gglongs [AT] 163.com

Bingnan Pei

College of Information Engineering
Dalian University
Dalian, China

Abstract—Multi-input multi-output (MIMO) radar waveform optimization is often sensitive to estimation errors and uncertainty in the initial parameters estimates (i.e., some prior information on the target of interest and scenario). Focusing on this issue, the robust waveform design, which attempts to systematically alleviate this sensitivity by explicitly incorporating a parameter uncertainty model in the optimization problem, is considered to improve the worst-case detection performance over a convex uncertainty model in this paper. To solve the complicated nonlinear robust optimization problem, an iterative algorithm is proposed to optimize the waveform covariance matrix (WCM) for maximizing the worst-case output signal-interference-noise-ratio (SINR). In the proposed algorithm, each iteration step can be reformulated as a semidefinite programming (SDP) problem, which can be solved very efficiently. Numerical results show that, compared to those of the non-robust method and uncorrelated waveforms, the worst-case detection performance can be improved considerably by the proposed method.

Keywords- Multi-input multi-output (MIMO) radar, robust waveform design, target detection, semidefinite programming (SDP)

I. INTRODUCTION

In recent years, multiple-input multiple-output (MIMO) techniques have received more and more attention from both the communication and radar communities [1]-[16]. MIMO radar can employ multiple transmitting elements to transmit arbitrary waveforms other than coherent waveforms in traditional phased-array radars [1]. Two categories of MIMO radar systems can be classified by the configuration of the transmitting and receiving antennas: (1) MIMO radar with widely separated antennas (e.g. [1]), and (2) MIMO radar with colocated antennas (e.g. [2]). For MIMO radar with widely separated antennas, the transmitting and receiving elements are widely spaced such that each views a different aspect of the target. In contrast, MIMO radar with colocated antennas, whose elements in transmitting and receiving arrays are close enough such that the target radar cross sections (RCSs) observed by MIMO radar are identical, can utilize the waveform diversity to increase the virtual aperture of the receiving array [2]. Accordingly, it has several advantages

including improved parameter identifiability [3], [4], and more flexibility for transmit beampattern design [5]-[12].

To improve the detection performance of MIMO radar, one way is detector design which was investigated in [13], [14]. In [13], C. Y. Chong et al. proposed the constant false alarm rate (CFAR) generalized likelihood ratio test-linear quadratic (GLRT-LQ) detector for MIMO radar in the scenario of non-Gaussian clutter. In [14], Q. He et al. derived GRLT moving target detectors for centralized MIMO and distributed MIMO radar.

Another way to improve the detection performance of MIMO radar is waveform optimization, which has been studied in [7]-[10]. In [7], H. Wang et al. designed the transmitted waveforms to improve the detection performance of MIMO space-time adaptive processing (STAP) by exploiting diagonal loading (DL) method with perfect target and clutter prior knowledge. However, the information about target and clutter used in the waveform optimization must be estimated with error in practice. Therefore, the robust waveform optimization for MIMO-STAP in the case of imperfect space-time steering vector prior knowledge is considered to improve the worst-case detection performance in [8]. In [9], a gradient based method is proposed to maximize the output signal-to-interference-plus-noise ratio (SINR) for improving the detection performance for extended target; unfortunately, it cannot guarantee nondecreasing SINR in each iteration step. In order to guarantee convergence, a new iterative algorithm is proposed in [10].

It is known that waveform optimization for improving the performance of MIMO radar usually depends on the initial parameter estimate (e.g., the target location, reflection coefficients, etc.) [7]-[10]. As a sequence, the optimized waveforms depend on these pre-assigned values. In practice, these parameters are estimated with errors, and hence they are uncertain. As illustrated by numerical examples in [7], the resultant output SINR, i.e., detection probability, is sensitive to these estimation errors and uncertainty in parameters, which is similar to that in the case of waveform design for improving the parameter estimation performance [5]. It means that the optimized waveforms based on a certain parameter estimate

can give a very low detection performance for another reasonable estimate.

In order to improve the worst-case detection performance, the problem of robust waveform design is addressed in this paper, which attempts to systematically alleviate the sensitivity by explicitly incorporating a parameter uncertainty model in the optimization issue. The WCM is optimized to obtain the best worst-case detection performance over a convex uncertainty set. An iterative algorithm is proposed to solve the optimization problem such that the worst-case performance can be improved. Each step in the proposed algorithm can be reformulated as a semidefinite programming (SDP) problem [17], [18], and hence it can be solved efficiently.

The rest of this paper is organized as follows. Section II introduces the MIMO radar model, and formulates the waveform optimization problem incorporating the convex uncertainty set of parameters. Section III proposes an iteration algorithm to improve the worst-case detection performance. Section IV shows the effectiveness of the proposed method via numerical examples. Finally, Section V concludes this paper.

Throughout the paper, matrices and vectors are denoted by boldface uppercase and lowercase letters, respectively. We use $\{\cdot\}^T$, $\{\cdot\}^*$, and $\{\cdot\}^H$ to represent the transpose, conjugate, and conjugate transpose, respectively. The symbol \otimes indicates the Kronecker product, \mathbf{I} denotes the identity matrix, and $\text{vec}\{\cdot\}$ is the vectorization operator stacking the columns of a matrix on top of each other. The notation $\|\mathbf{A}\|_F$ stands for the Frobenius norm of the matrix. Denote by $\text{tr}\{\cdot\}$, $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ the trace, the real and imaginary part of a matrix, respectively. Finally, the notation $\mathbf{A} \circ \mathbf{B}$ means that $\mathbf{B} - \mathbf{A}$ is positive semidefinite.

II. PROBLEM FORMULATION

Consider a MIMO radar system with M_t transmitting elements and M_r receiving elements. Let $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_{M_t}]^T \in \mathbb{C}^{M_t \times L}$ be the transmitted waveform matrix, where $\mathbf{s}_i \in \mathbb{C}^{L \times 1}$, $i = 1, 2, \dots, M_t$ denotes the discrete-time baseband signal of the i th transmitting element with L being the number of snapshots. Under the assumption that the probing signals are narrowband and the propagation is non-dispersive, the signals received by MIMO radar can be expressed as (see, e.g., [5]):

$$\mathbf{Y} = \beta \mathbf{a}(\boldsymbol{\theta}) \mathbf{v}^T(\boldsymbol{\theta}) \mathbf{S} + \mathbf{Z}, \quad (1)$$

where the columns of $\mathbf{Y} \in \mathbb{C}^{M_r \times L}$ are the collected data snapshots, β is the complex amplitudes proportional to the RCS of the target of interest, and $\boldsymbol{\theta}$ denotes those location parameters. The parameters β and $\boldsymbol{\theta}$ need to be estimated

from the received signal \mathbf{Y} . The term \mathbf{Z} is the noise plus interference, whose columns can be assumed to be independent and identically distributed circularly symmetric complex Gaussian random vectors with mean zero and an unknown covariance denoting by \mathbf{Q} [5]. Also, $\mathbf{a}(\boldsymbol{\theta})$ and $\mathbf{v}(\boldsymbol{\theta})$ denote, respectively, the receiving and transmitting steering vectors for the target located at $\boldsymbol{\theta}$, which can be described as

$$\begin{aligned} \mathbf{a}(\boldsymbol{\theta}) &= [e^{j2\pi f_0 \tau_1(\boldsymbol{\theta})}, e^{j2\pi f_0 \tau_2(\boldsymbol{\theta})}, \dots, e^{j2\pi f_0 \tau_{M_r}(\boldsymbol{\theta})}]^T \\ \mathbf{v}(\boldsymbol{\theta}) &= [e^{j2\pi f_0 \tilde{\tau}_1(\boldsymbol{\theta})}, e^{j2\pi f_0 \tilde{\tau}_2(\boldsymbol{\theta})}, \dots, e^{j2\pi f_0 \tilde{\tau}_{M_t}(\boldsymbol{\theta})}]^T, \end{aligned} \quad (2)$$

where f_0 represents the carrier frequency. $\tau_m(\boldsymbol{\theta})$, $m = 1, 2, \dots, M_r$ is the propagation time from the target located at $\boldsymbol{\theta}$ to the m th receiving element, and $\tilde{\tau}_n(\boldsymbol{\theta})$, $n = 1, 2, \dots, M_t$ is the propagation time from the n th transmitting element to the target.

By employing $\mathbf{S}^H (\mathbf{S} \mathbf{S}^H)^{-1/2}$ as the matched-filter bank at the receiver to get the sufficient statistics for target detection, and the output of the filter can be stacked in a $M_r M_t \times 1$ vector as

$$\mathbf{y} = \beta \mathbf{T} \mathbf{h} + \text{vec}(\tilde{\mathbf{Z}}), \quad (3)$$

where $\mathbf{y} = \text{vec}(\mathbf{Y} \mathbf{S}^H (\mathbf{S} \mathbf{S}^H)^{-1/2})$, $\mathbf{T} = (\mathbf{S} \mathbf{S}^H)^{1/2} \otimes \mathbf{I}_{M_r}$, $\mathbf{h} = \mathbf{a}(\boldsymbol{\theta}) \otimes \mathbf{b}(\boldsymbol{\theta})$, $\tilde{\mathbf{Z}} = \mathbf{Z} \mathbf{S}^H (\mathbf{S} \mathbf{S}^H)^{-1/2}$.

Considering the problem of detecting a target in the cell under test, with the model in (3), this issue can be formulated in terms of the following binary hypotheses test

$$\begin{cases} H_0: & \mathbf{y} = \text{vec}(\tilde{\mathbf{Z}}) \\ H_1: & \mathbf{y} = \beta \mathbf{T} \mathbf{h} + \text{vec}(\tilde{\mathbf{Z}}) \end{cases}. \quad (4)$$

Under the assumption of \mathbf{Z} above, the joint probability density function (pdf) of the received vectors conditioned on the phase of β and hypotheses can be written as

$$f(\mathbf{y} | \phi, H_0) = \frac{1}{\pi^{M_r M_t} |\mathbf{I} \otimes \mathbf{Q}|} e^{-\mathbf{y}^H (\mathbf{I} \otimes \mathbf{Q})^{-1} \mathbf{y}}, \quad (5)$$

$$f(\mathbf{y} | \phi, H_1) = \frac{1}{\pi^{M_r M_t} |\mathbf{I} \otimes \mathbf{Q}|} e^{-(\mathbf{y} - \beta \mathbf{T} \mathbf{h})^H (\mathbf{I} \otimes \mathbf{Q})^{-1} (\mathbf{y} - \beta \mathbf{T} \mathbf{h})}. \quad (6)$$

If ϕ is uniformly distributed in $[0, 2\pi]$ [19], with (5) and (6), the generalized likelihood ratio test (GRLT) detector for the binary hypotheses test can be given by

$$\left| \mathbf{y}^H (\mathbf{I}_{M_r} \otimes \mathbf{Q}^{-1}) \mathbf{T} \mathbf{h} \right|_{H_0}^2 \underset{H_0}{\overset{H_1}{\gtrless}} \eta, \quad (7)$$

where η' is the detection threshold set according to a desired value of the false alarm probability (P_{fa}). In the case of nonfluctuating target, an analytical expression of the detection probability (P_d), for a given value of P_{fa} , can be expressed as [19]:

$$P_d = Q(\sqrt{2|\beta|^2 \text{tr}((\mathbf{T}\mathbf{h})^H (\mathbf{I} \otimes \mathbf{Q})^{-1} \mathbf{T}\mathbf{h})}, \sqrt{-2 \ln P_{fa}}), \quad (8)$$

where $Q(\cdot, \cdot)$ denotes the Marcum Q function of order 1.

By using some matrix manipulations, (8) can be rewritten as

$$P_d = Q(\sqrt{2|\beta|^2 \text{tr}(\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H} \mathbf{R}_s)}, \sqrt{-2 \ln P_{fa}}), \quad (9)$$

where $\mathbf{H} = \mathbf{a}(\boldsymbol{\theta}) \mathbf{v}^T(\boldsymbol{\theta})$ is the target channel matrix at the considered range bin.

It is noted that the output signal-noise ratio (SNR) can be obtained as [19]:

$$SNR = |\beta|^2 \text{tr}(\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H} \mathbf{R}_s). \quad (10)$$

Given P_{fa} , it can be seen from (9) and (10) that P_d is an increasing function of the output SNR. As a sequence, maximization of P_d is tantamount to maximization of the output SNR.

It can be seen from (10) that SNR is a function of the location $\boldsymbol{\theta}$ as well as the noise plus interference. In practice, these parameters are estimated with errors. Hence, the optimized waveforms based on the SNR employing a parameter estimate can give a very low accuracy for another reasonable estimate, which can be seen from numerical examples in [5], [7].

Here, we assume that the target channel matrix can be modeled as:

$$\mathbf{H} = \tilde{\mathbf{H}} + \boldsymbol{\delta}, \quad (11)$$

where \mathbf{H} and $\tilde{\mathbf{H}}$ denote, respectively, the actual and corresponding presumed channel, and $\boldsymbol{\delta}$ is the error of \mathbf{H} , which belongs to the set

$$U = \{ \boldsymbol{\delta} \mid \|\boldsymbol{\delta}\| \leq \sigma \}. \quad (12)$$

The robust waveform design for target detection can now be briefly stated as follows: Optimize the WCM to maximize the worst-case SNR over the convex set U under the constraints about the WCM. Based on (10) and (12), this optimization problem can be illustrated as

$$\begin{aligned} \max_{\mathbf{R}_s} \quad & \min_{\boldsymbol{\delta}} \quad SNR \\ \text{s.t.} \quad & \boldsymbol{\delta} \in U \\ & \text{tr}(\mathbf{R}_s) = LP \\ & \mathbf{R}_s \succeq \mathbf{0} \end{aligned} \quad (13)$$

where P denotes the total transmitted power. The third constraint holds due to the power transmitted by any transmitting element is more than or equal to zero in practice.

III. PROPOSED ITERATIVE METHOD

In this section, we will show how to solve the robust optimization problem to obtain the better worst-case detection performance.

We now treat the inner optimization problem firstly. It can be seen the optimization variables in the problem above are complex. To facilitate the solution of (13), we can convert it to a real-valued form in Appendix, which is shown as

$$\begin{aligned} \min_{\boldsymbol{\delta}_R} \quad & |\beta|^2 \text{tr}(\mathbf{H}_R^T \mathbf{Q}_R^{-1} \mathbf{H}_R \mathbf{R}_{R,S}) \\ \text{s.t.} \quad & \|\boldsymbol{\delta}_R\|_F \leq \gamma \end{aligned} \quad (14)$$

where \mathbf{H}_R , $\mathbf{R}_{R,S}$, and $\boldsymbol{\delta}_R$ are defined in **Error! Reference source not found.**, γ in **Error! Reference source not found.**, and

$$\mathbf{Q}_R = \begin{bmatrix} \mathbf{Q}_x & -\mathbf{Q}_y \\ \mathbf{Q}_y & \mathbf{Q}_x \end{bmatrix}, \quad (15)$$

$$\mathbf{Q}_x = \text{Re}(\mathbf{Q}), \mathbf{Q}_y = \text{Im}(\mathbf{Q}). \quad (16)$$

It is obvious that the term $\text{tr}(\mathbf{H}_R^T \mathbf{Q}_R^{-1} \mathbf{H}_R \mathbf{R}_{R,S})$ is convex with respect to $\boldsymbol{\delta}_R$ [20]. Denoting by λ_{max} the largest eigenvalue of $\mathbf{H}_R^T \mathbf{Q}_R^{-1} \mathbf{H}_R \mathbf{R}_{R,S}$, (14) can be equivalently represented as

$$\begin{aligned} \min_{\boldsymbol{\delta}_R} \quad & \lambda_{max} \\ \text{s.t.} \quad & \mathbf{T}^T \mathbf{H}_R^T \mathbf{Q}_R^{-1} \mathbf{H}_R \mathbf{T} \preceq \lambda_{max} \mathbf{I}, \\ & \|\boldsymbol{\delta}_R\|_F \leq \gamma \end{aligned} \quad (17)$$

where $\mathbf{T} = \mathbf{R}_{R,S}^{1/2}$, i.e., \mathbf{T} is the square root of $\mathbf{R}_{R,S}$ [21]. The constraints in (17) can be reformulated as a linear matrix inequalities (LMI) with respect to $\boldsymbol{\delta}_R$, relying on the following lemma [21, pp.472]:

Lemma 1 (Schur's Complement): Let $\mathbf{Z} = \begin{bmatrix} \mathbf{A} & \mathbf{B}^H \\ \mathbf{B} & \mathbf{C} \end{bmatrix}$ be a Hermitian matrix with $\mathbf{C} \succ \mathbf{0}$, then $\mathbf{Z} \succeq \mathbf{0}$ if and only if

$\Delta\mathbf{C} \pm \mathbf{0}$, where $\Delta\mathbf{C}$ is the Schur complement of \mathbf{C} in \mathbf{Z} and is given by $\Delta\mathbf{C} = \mathbf{A} - \mathbf{B}^H \mathbf{C}^{-1} \mathbf{B}$.

By using Lemma 1, the problem can be recast as an SDP shown as

$$\begin{aligned} \min_{\delta_R} \quad & \lambda_{max} \\ \text{s.t.} \quad & \begin{bmatrix} \gamma^2 \mathbf{I} & \delta_R^T \\ \delta_R & \mathbf{I} \end{bmatrix} \pm \mathbf{0} \\ & \begin{bmatrix} \lambda_{max} \mathbf{I} & \mathbf{T}^T (\tilde{\mathbf{H}}_R + \delta_R)^T \\ (\tilde{\mathbf{H}}_R + \delta_R) \mathbf{T} & \mathbf{Q}_R \end{bmatrix} \pm \mathbf{0} \end{aligned} \quad (18)$$

Substituting δ_R obtained from (18) into (13), \mathbf{R}_S can be solved by an SDP

$$\begin{aligned} \min_{\mathbf{R}_S, t} \quad & -t \\ \text{s.t.} \quad & |\beta|^2 \text{tr}(\mathbf{H}^H \mathbf{Q}^{-1} \mathbf{H} \mathbf{R}_S) \geq t, \\ & \text{tr}(\mathbf{R}_S) = LP \\ & \mathbf{R}_S \pm \mathbf{0} \end{aligned} \quad (19)$$

where t is an auxiliary variable.

So far, we know how to solve δ for fixed \mathbf{R}_S , and \mathbf{R}_S for fixed δ . We can iteratively optimize δ and \mathbf{R}_S . Similar to Algorithm 3 proposed in [10], an iterative algorithm is proposed to improve the worst-case detection performance, which is shown as follows.

Algorithm: Given an initial value of the WCM, δ and \mathbf{R}_S can be optimized by repeating the following steps:

- a) Solve (18) to obtain the optimum δ .
- b) Solve (19) to obtain \mathbf{R}_S .

Go to step 1 until the SNR increase becomes insignificant.

Using many well-known algorithms (see, e.g., [17]) for solving SDP problems, the problems (18), (19) can be solved very efficiently. In the following numerical example, the optimization toolbox in [18] is used for these problems. It is noticed that the proposed method can only obtain the WCM other than the ultimate transmitted waveforms. In practice, the ultimate waveforms can be asymptotically synthesized by using the method in [22].

IV. NUMERICAL EXAMPLES

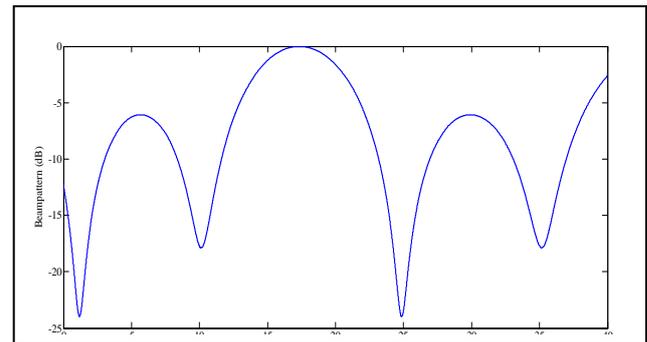
In this section, we assess the performance of the proposed method compared to the non-robust method that can be calculated by (10), and uncorrelated waveforms that can be generated by using Hadamard codes [16].

Consider a MIMO radar system with $M_t = 4$ transmitting elements and $M_r = 4$ receiving elements. We use the MIMO radar (0.5, 0.5) in the following examples, where the parameters specifying each radar system are the inter-element spacing of the transmitter and receiver (in units of wavelengths), respectively. The number of snapshots is $L = 256$. The array signal-to-noise ratio (ASNR) varying from -15 to 20 dB in the following examples is defined as $PM_t M_r / \sigma_w^2$, where P stands for the total transmitted power, and σ_w^2 denotes the variance of the additive white thermal noise. There is a strong jammer at 10° with an array-interference-to-noise ratio (AINR) defined as the product of the incident interference power and M_r divided by σ_w^2 , equal to 60 dB. In the following numerical examples, there is only one target with unit amplitude at $\theta = 20^\circ$ in the considered range bin.

It is known from Section II that the output SNR must be estimated using the initial location parameter estimate. There are many methods for estimating this parameter (see, e.g., [23] and the reference therein for more details).

In the following examples, we examine the effectiveness of the proposed method in the case of existing the initial angle estimate error. In this case, it is assumed that the initial angle estimate has an uncertainty $\Delta\theta = [-2^\circ, 2^\circ]$.

Fig. 1 shows the optimal transmit beampattern with ASNR=10 dB. It can be seen that the peak of the transmit beampattern is placed around the target location, which means that the worst-case detection performance in the convex uncertainty can be improved by the proposed method. Moreover, it can be seen that a notch is placed almost at the

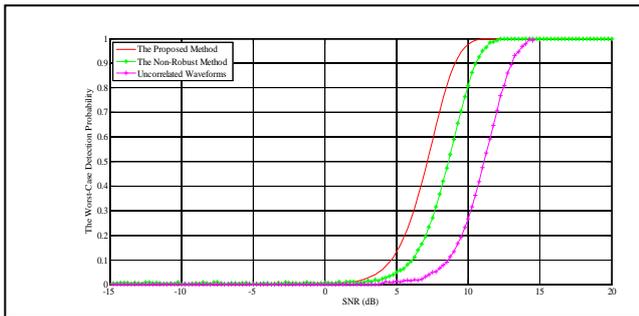


jammer location.

Figure 1. Optimal transmit beampattern with ASNR=10 dB in the case of the initial angle estimate error.

In Fig. 2, the worst-case detection probability obtained by the proposed method versus ASNR in the case of the initial angle estimate error is shown, as compared to that of uncorrelated waveforms and the non-robust method, in the

case of $P_{fa} = 10^{-6}$. It is obvious that the detection probabilities obtained by three methods increase as the increase of ASNR. Moreover, one can observe that the transmitted waveforms obtained by the proposed method have a better worst-case



detection performance as compared to those proposed by the non-robust method and uncorrelated waveforms.

Figure 2. The worst-case detection probability for $P_{fa} = 10^{-6}$ versus ASNR, as well as that of uncorrelated waveforms and the non-robust method, in the case of the initial angle estimate error.

V. CONCLUSIONS

In this paper, the problem of robust waveform optimization, which improves the target detection performance of MIMO radar by explicitly incorporating the uncertainty in parameters into the optimization model, has been investigated. An iterative algorithm has been proposed to solve the sophisticated nonlinear problem, each step in which is solved resorting to a convex relaxation that belongs to the SDP class. Numerical examples have shown that the proposed iterative algorithm improves the worst-case detection probability very obviously compared to uncorrelated waveforms as well as the non-robust method. Therefore, the transmitted waveforms obtained by the proposed algorithm can improve the overall detection performance for MIMO radar.

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REFERENCES

- [1] E. Fishler, A. Haimovich, R. Blum, D. Chizhik, L. Cimini, and R. Valenzuela, "MIMO radar: an idea whose time has come," [Proceedings of the IEEE Radar Conference, Newark, NJ, USA, 26–29, pp. 71–78, Apr. 2004].
- [2] J. Li, and P. Stoica, "MIMO radar with colocated antennas," IEEE Signal Processing Magazine, vol. 24, pp. 106–114, Sept. 2007.
- [3] J. Li, P. Stoica, L. Xu, and W. Roberts, "On parameter identifiability of MIMO radar," IEEE Trans. on Signal Processing Lett., vol. 14, pp. 968–971, 2007.

- [4] Hongyan Wang, Guisheng Liao, Yong Wang, and Xiangyang Liu, "On parameter identifiability of MIMO radar with waveform diversity," Signal Processing, vol. 91, pp. 2057–2063, 2011.
- [5] J. Li, L. Xu, P. Stoica, K. W. Forsythe, and D. W. Bliss, "Range Compression and Waveform Optimization for MIMO Radar: A Cramer-Rao Bound Based Study," IEEE Trans. on Signal Processing, vol. 56, pp. 218–232, Jan. 2008.
- [6] Daniel Rabideau, "MIMO radar Waveforms and Cancellation Ratio," IEEE Trans. on Aerospace and Electronic Systems, vol. 48, pp. 1167–1178, 2012.
- [7] Hongyan Wang, Guisheng Liao, Jun Li, and Hui Lv, "Waveform Optimization for MIMO-STAP to Improve the Detection performance," Signal Processing, vol. 91, pp. 2690–2696, 2011.
- [8] Hongyan Wang, Guisheng Liao, Jun Li, and Wangmei Guo, "Robust Waveform Design for MIMO STAP to Improve the Worst-Case Detection Performance," EURASIP J. Adv. Signal Processing, 2013:52, Mar. 2013.
- [9] B. Friedlander, "Waveform Design for MIMO Radars," IEEE Trans. on Aerospace and Electronic Systems, vol. 43, pp. 1227–1238, Jul. 2007.
- [10] C. Y. Chen, and P. P. Vaidyanathan, "MIMO Radar Waveform Optimization with Prior Information of the Extended Target and Clutter," IEEE Trans. on Signal Processing, vol. 57, pp. 3533–3544, 2009.
- [11] Yongchao Wang, Xu Wang, Hongwei Liu, and Zhiquan Luo, "On the Design of Constant Modulus Probing Signals for MIMO Radar," IEEE Trans. on Signal Processing, vol. 60, pp. 4432–4438, 2012.
- [12] Jun Liu, Hongbin Li, and Braham Himed, "Joint Optimization of Transmit and Receive Beamforming in Active Arrays," IEEE Trans. on Signal Processing Lett., vol. 21, pp. 39–42, Jan. 2014.
- [13] C. Y. Chong, Frédéric Pascal, Jean-Philippe Ovarlez, and Marc Lesturgie, "MIMO radar detection in non-Gaussian and heterogeneous clutter," IEEE Journal of Selected Topics in Signal Processing, vol. 4, pp. 115–126, Feb. 2010.
- [14] Q. He, N. H. Lehmann, R. S. Blum, and A. M. Haimovich, "MIMO radar moving target detection in homogeneous clutter," IEEE Trans. on Aerospace and Electronic Systems, vol. 46, pp. 1290–1301, Jun. 2010.
- [15] Pier Francesco Sannarino, Christopher J. Baker, and Hugh D. Griffiths, "Frequency Diverse MIMO Techniques for radar," IEEE Trans. on Aerospace and Electronic Systems, vol. 49, pp. 201–222, 2013.
- [16] J. G. Proakis, Digital Communications, 4th ed. New York: McGraw Hill, 2001.
- [17] A. Ben-Tal, and A. Nemirovski, Lectures on Modern Convex Optimization, ser. Optimization. Philadelphia, PA: MPS-SIAM, 2001.
- [18] J. Lofberg, "YALMIP: A Toolbox for Modeling and Optimization in MATLAB," [Proceedings of the CACSD Conference, Taipei, Taiwan, pp. 284–289, 2004].
- [19] J. S. Goldstein, I. S. Reed, and P. A. Zulch, "Multistage partially adaptive STAP CFAR detection algorithm," IEEE Trans. Aerosp. Electron. Syst., vol. 35, pp. 645–661, Apr. 1999.
- [20] Seung-Jean Kim, Alessandro Magnani, Almir Mutapcic, Stephen P. Boyd, and Zhi-Quan Luo, "Robust Beamforming via Worst-Case SINR Maximization," IEEE Trans. On Signal Processing, vol. 56, pp. 1539–1547, Apr. 2008.
- [21] R. A. Horn, and C. R. Johnson, Matrix Analysis, Cambridge, U.K.: Cambridge Univ. Press, 1985.
- [22] P. Stoica, J. Li, and X. Zhu, "Waveform Synthesis for Diversity-Based Transmit Beampattern Design," IEEE Trans. On Signal Processing, vol. 56, pp. 2593–2598, 2008.
- [23] L. Xu, J. Li, and P. Stoica, "Adaptive techniques for MIMO radar," [The 4th IEEE Workshop on Sensor Array and Multi-Channel Processing, Waltham, MA, Jul. 2006].