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A Lower Bound for Codes Correcting Low Density Closed-Loop Burst

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Abstract—**In this paper, we present a lower bound for low density closed loop burst error correcting code in two sub-blocks of length n_1 and n_2 of the total code length n $(n = n_1 + n_2)$.**

Keywords-Open loop burst, Closed loop burst, Parity check matrix,Syndromes, Error correction.

I. INTRODUCTION

In many communication systems error occurs predominantly in the form of a burst. An error which occurs among a specific number of consecutive positions is referred to as a burst error. The definition of a burst or an open loop burst runs as, "A burst of length b is a vector whose only nonzero components are confined to some b consecutive positions, the first and the last of which are nonzero"'.

Burst error correcting codes have been developed with a view to correct such errors. There are many situations where error occurs in the form of a burst but not all the digits inside the burst get corrupted. Such errors are referred to as low density/high density burst errors. In low density burst error correcting code, the code corrects burst of length b or less with weight w or less. On the other hand in high density case, the code corrects burst of length b or less with weight w or more $(w \le b)$. So it is always useful to study the error correcting capabilities of burst codes with weight constraints. Such codes are studied by Sharma and Dass (1974), Dass (1975), Muttoo and Tyagi (1981), Luigia and Dass (1996) and many others. There is also a different kind of burst called a closed loop burst defined by Campopiano (1962) which runs as, "If $2 \le b \le \frac{n+1}{2}$, then $(a_1, a_2, \ldots, a_n)$, is called a closed loop burst of length b . Whenever there is a i such that $1 \le i \le b-1$, $a_i a_{n-b+i+1} \ne 0$, $a_{i+1} = a_{i+2} = \cdots = a_{n-b+i} = 0$ ".

Clearly the definition of closed loop burst includes the definition of an open loop burst; therefore while considering the class of closed loop burst of length b or less, the set of open loop burst of length b or less is automatically included.

In this paper, we deal with the problem of correcting a closed loop burst of length b or less with weight w or less $(w \le b)$ in two sub-blocks of length n_1 and n_2; $(n = n_1 + n_2)$. The study of Blockwise burst error correcting codes has been found useful for faster communication system and with a better rate of transmission as they consume lesser time in error detection/correction.

II. LOWER BOUND

Theorem 1. *The number of parity check digits in an (n,k) linear code that corrects all closed loop burst of length b_1 or less with weight w or less $(w \le b_1)$ in first sub-block of length n_1 and all closed loop burst of length b_2 or less with weight w in second sub-block of length n_2 is at least*

$$q^{(n-k)} \ge 1 + (n_1+n_2)(q-1) + \left[n_1 \sum_{i=2}^{b_1}\sum_{j=2}^{w} \binom{i-2}{j-2} + n_2 \sum_{i=2}^{b_2}\sum_{j=2}^{w} \binom{i-2}{j-2} \right] (q-1)^j . \qquad (1)$$

Proof. Let there be an (n,k) linear code vector $\mathrm{GF}(q)$ that correct all closed loop low-density burst based on counting the number of errors of specific type.

Since the code is capable of correcting all closed loop burst of length b_1 or less with weight w or less in first sub-block of length n_1, should be in different cosets of the standard array, their number is

$$\begin{aligned} & n_1(q-1) + \sum_{i=2}^{w} n_1 (q-1)^2 + n_1 \sum_{i=2}^{b_1}\sum_{j=2}^{w} \binom{i-2}{j-2} (q-1)^j \\ & = n_1(q-1) + n_1 \sum_{i=2}^{b_1}\sum_{j=2}^{w} \binom{i-2}{j-2} (q-1)^j . \end{aligned} \qquad (2)$$

Similarly, as the code is capable of correcting all closed loop burst of length b_2 or less with weight w or less in second sub-block of length n_2 should be in different cosets of the standard array, their number is

$$n_2(q-1)+\sum_{i=2}^{w} n_2(q-1)^2 + n_2\sum_{i=2}^{b_2}\sum_{j=2}^{w}\binom{i-2}{j-2}(q-1)^j$$
$$= n_2(q-1)+n_2\sum_{i=2}^{b_2}\sum_{j=2}^{w}\binom{i-2}{j-2}(q-1)^j . \tag{3}$$

Thus the total number of errors including the zero vector from (2) and (3) is

$$1+n_1(q-1)+n_1\sum_{i=2}^{b_1}\sum_{j=2}^{w}\binom{i-2}{j-2}(q-1)^j$$
$$+n_2(q-1)+n_2\sum_{i=2}^{b_2}\sum_{j=2}^{w}\binom{i-2}{j-2}(q-1)^j$$
$$=1+(n_1+n_2)(q-1)$$
$$+\left[n_1\sum_{i=2}^{b_1}\sum_{j=2}^{w}\binom{i-2}{j-2}+n_2\sum_{i=2}^{b_2}\sum_{j=2}^{w}\binom{i-2}{j-2}\right](q-1)^j .$$

Since, there must be at least this number of cosets and number of possible cosets is q^{n-k}, so

$$q^{(n-k)} \geq 1+(n_1+n_2)(q-1)$$
$$+\left[n_1\sum_{i=2}^{b_1}\sum_{j=2}^{w}\binom{i-2}{j-2}+n_2\sum_{i=2}^{b_2}\sum_{j=2}^{w}\binom{i-2}{j-2}\right](q-1)^j . \tag{4}$$

III. DISCUSSION

Taking $n_2=0$, $b_1=b$, the bound obtained in (1) reduces to

$$q^{(n-k)} \geq 1+n(q-1)+\left[n\sum_{i=2}^{b}\sum_{j=2}^{w}\binom{i-2}{j-2}\right](q-1)^j \tag{5}$$

which coincides with a result due to Muttoo and Tyagi (1992) obtained for a code that corrects low density closed loop burst errors.

Lastly, for $w=b$, the bound obtained in (1), becomes

$$\log_q\left\lfloor 1+n(q-1)q^{b-1}\right\rfloor$$

which is a result by Campopiano (1962).

Example. Consider a $(6+7,5)$ binary linear code with parity-check matrix

$$\begin{bmatrix} 1&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&1&0&0&0&0&0&0&1&1&0&0&0\\ 0&0&1&0&0&0&0&0&1&1&1&0&0\\ 0&0&0&1&0&0&0&0&1&1&1&1&0\\ 0&0&0&0&1&0&0&0&0&1&1&1&1\\ 0&0&0&0&0&1&0&0&0&0&1&1&1\\ 0&0&0&0&0&0&1&0&0&0&0&1&1\\ 0&0&0&0&0&0&0&1&0&0&0&0&1 \end{bmatrix}_{8\times(6+7)}$$

This matrix has been constructed by the synthesis procedure outlined in the Theorem 1, taking $q=2$, $b_1=2$, $b_2=3$, $w=2$, $n_1=6$, $n_2=7$. The code which is the null space of matrix given above corrects all closed loop burst of length 2 or less with weight 2 or less in the first block of length 6 and all closed loop burst of length 3 or less with weight 2 or less in the second block of length 7. It follows from the Table 1 that all the error vectors and their corresponding syndromes which can be seen to be all distinct and non-zero.

TABLE I.

Error Pattern	***Syndromes***
1st block	
110000 0000000	11000000
011000 0000000	01100000
001100 0000000	00110000
000110 0000000	00011000
000011 0000000	00001100
100000 0000000	10000000
010000 0000000	01000000
001000 0000000	00100000
000100 0000000	00010000
000010 0000000	00001000
000001 0000000	00000100
100001 0000000	10000100
2nd sub block	
000000 1110000	11110011
000000 0111000	10001001
000000 0011100	10110100
000000 0001110	01011010
000000 0000111	00101101
000000 1010000	11110010
000000 0101000	01111001
000000 0010100	11001100
000000 0001010	01100110
000000 0000101	00110011
000000 1100000	00000011
000000 0110000	11110001
000000 0011000	10001000
000000 0001100	01000100
000000 0000110	00100010
000000 0000011	00010001
000000 1000000	00000010
000000 0100000	00000001
000000 0010000	11110000
000000 0001000	01111000
000000 0000100	00111100
000000 0000010	00011110
000000 0000001	00001111
000000 1000011	10010001
000000 1100001	00001100
000000 1000010	00011100
000000 0100001	00001110
000000 1000001	00001101

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