Minimizing Problem of Power Loss and its Solving Method on the Radial Distribution Networks with Weighting Load

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Abstract -
The power network of the smallest power loss cost is the radial one, on which each load is directly supplied from the substations. In this paper, we discussed the method for decreasing minimumly the power loss by smoothly increasing the weighted load of stations on the radial distribution network. Here we formalized the determination problem of power supply range of substations on radial distuibution network to the non-_linear programming model, and proposed the method that replaces the given model by linear programming one and its sloving method.

Keywords- the weighted voronoi diagram, power distribution planning, the non linear programming.

I. INTRODUCTION

In traditional power distribution planning(PDP) on the radial distribution network, the models of minimizing theoptimal location and size, power supply range of stations and the investment cost are proposed[1-3].

Avobe models are defined of large-scale, nonlinear integer optimization problem. Typical methods to solve this problem are Mathematical programming methods which contain the Mixed integer linear programming[4,5] and Non linear programming[6,7], Multi-objective programming[8], Dynamic programming[9] etc., and Soft calculation methods which contain the Genetic algorithm[10-14], Tabu search[15], and Particle swarm optimization[16], Evolutionary algorithm[17], Ant colony system[18], Bacterial foraging[19] etc., Geometric calculation methods which contain an ordinary Voronoi diagram, the weighted Voronoi diagram[21-23]. These methods have merits and demerits respectively. Mathematical programming methods had strict optimality but could hardly get a feasible optimized solution when faced with complex and large –scale problems[24]. Soft calculation methods provide near-optimal solutions for large-scale PDP problems but there is no guarantee that they will find a global optimal solution [24].

Geometric calculation methods had good computational stability and can reduce the computing times; however, there is no guarantee that they will find better solution than Soft calculation method [25].

In [26] improved weight was calculated by adding an adaptive control process of load ratio, the substation site was determined by using the Voronoi diagram and then the power supply which could get a reasonable load rate for each substation is calculated.

But the function expressing the improved weight is non linear. Therefore, there isn't guarantee that it's an optimal solution.

Normal determination method of power supply range is one that determine the station's locations and power supply range so that sum of station's construction cost and annual management cost is minimized under the supposion of giving the power load distribution, numbers, capacity and upper limit of power supply range of newly constructed stations, so it is formalized such below. [26].

$$\sum_{k\in J_i} p_{ik} \le s_i r_i \cos\varphi,\ i=\overline{1,\ N}$$

$$J_1 \cup J_2 \cup \cdots \cup J_N = J$$

$$l_{ik} \le R_i,\ i=\overline{1,\ N},\ k\in J_i$$

$$\sum_{i=1}^{n}\left(f(s_i)\frac{r_0(1+r_0)^{m_0}}{(1+r_0)^{m_0}-1} + u(s_i)\right) +$$

$$+\alpha\left(\frac{r_0(1+r_0)^{m_1}}{r_0(1+r_0)^{m_1}-1}\right)\sum_{i=1}^{N}\sum_{k\in J_i} l_{ik} + \beta\sum_{i=1}^{N}\sum_{k\in J_i} p_{ik}^2 l_{ik} \Rightarrow \min .$$

, here N is the total number of existing and newly constructed substations, n is the number of newly constructed substations, $f(s_i)$ is the intestment cost of substation i, $u(s_i)$ is the operation cost of newly constructed substation i , s_i is the capacity of substation i, r_i is the load rate of substation i, J_i is the load collection carried by substation i, J is the collection of all load points, l_{ik} is the length of feeder k

added from substation *i*, p_{ik} is the load (active power) carried by feeder k of substation *i*, m_0 is the depreciation years of the substations, m_1 is the depreciation years of the substations' low side, r_0 is the decount rate, $\cos\varphi$ is the power-factor (ration of effective power and total power), R_i is the limit of the power supply radius of substation, α is the investment cost per unit length of feeder, $\beta = \frac{\beta_1\beta_2\beta_3}{U^2}\cos^2\varphi$ is a feeder loss conversion factor, β_1 is the line resistance per unit length , β_2 is unit energy consumption discount factor, β_3 is the line loss hours per year, *U* is the line voltage of the substation low- voltage side.

The main variabls of above model are the power supply ranges R_i , but it's difficult to get them.

If the power supply radius of each substation is determinated, the corresponding power supply ranges are also determinated, thus investment, management and loss costs are determinated.

So the determination of the power supply range is a key of determinating power supply range.

Because some variables in the problem of determinating power supply range are non_negative integers, this problem becomes to the mixed integer non linear programming.

In [26], after location of substations based on their weighted voronoi diagram(WVD) by using the transportation model, the power supply radius of each substation is determinated.

The cost of [26]'s method is lower than the conventional algorithm.

This paper's method are also much faster than the conventional algorithm.

The method of above paper determined the approximate WVD and the power supply range by solving the transportation model .

Because WVD must be improved so that the difference between the supply capacity of substations and the arriving power of load point is minimum, power supply and WVD must be discussed on one system.

But in the method of [26] improvement of WVD and power supply are individually discussed, thus it can't guarantee that the given solution is optimal.

Nowadays in several countries, the power networks are constructed by radial networks for the minimum of power loss and the investment cost, and existing power networks are also reconstructed as branch networks like radial ones.

In this paper, we proposed the following problems to minimize the power loss of the power distribution network based on radial one.

1. Propose the mathematical model and research its properties to eliminate the power loss and evenly increase the weighted load of substations.
2. Research the calculating method to get the optimical point of minimizing the power loss in the radial network.
3. Check the efficiency of this method in several regions.

II. POWER LOSS MINIMIZING PROBLEM ON THE POWER NETWORK

2.1. weighted load of substations

The power network of the smallest power loss cost is the radial one, on which each load point is directly supplied from the substations.

Even on the branch type network, power supply is first discussed on the radial one and based on that, the power supply lines are constructed in branch one so that the line cost is low.

Power loss quantity on the power network mainly consists of power loss quantity on line and supply capacity loss of the substations which is expressed by the difference between power supply capacity and power quantity arriving to load point.

Because the power loss of line is expressed by the difference between the supplying power of substation to load point and the arriving power of load point, both the capacity loss of substations and loss quantity on line can be decreased at same time if the weighted load which is expressed by the ratio of the latter and the former is evenly increased.

The power supply capacity loss is *a-c* and the line loss is *b-c* when *a* is the sum of the substations's power capacity, *b* is the sum of the supplying power and *c* is the arriving power capacity to the load point.

The relationship is as following

Therefore, the line loss can decrease if supply capacity loss of substations decrease.

Thus, the minimizing problem of power loss is the weight decision one on the supposition that the weight of WVD is the weighted load of substations.

Suppose that s_i is the maximum power capacity of range D on the plane, $\cos\varphi_i$ is the power factor and there are m substations of load rate γ_i and n load points of need power β_j .

Let's M is the collection of substations and *N* is the collection of load points. ($|M|=m,\ |N|=n$).

Then the weighted voronoi diagram $V(w_i)$ of substation i is as following.

$$V(w_i) = \{ j \in N \mid w_i l_{ij} \le w_k l_{kj},\ k \in M \setminus \{i\}\},\ i \in M\ ,$$

where l_{ij} is the (Euclidean) distance between substation *i* and load point j, w_i is the weighted load of substation *i.*

Especially, the weighted voronoi diagram is the generalization of the usual voronoi diagrom because the weighted voronoi diagram $V(w_i)$ of substation i is the same as the usual voronoi diagram if $w_i = 1\ (i \in M)$.

Weighted load w_i of substation i expresses the balance relation between the supply capacity $s_i \gamma_i \cos\varphi_i$ of substation i and the arriving power v_i, then it is formed as following,

$$w_i = \frac{v_i}{s_i \gamma_i \cos\varphi_i},$$

where s_i is the capacity of substation i, r_i is the maximum load rate of substation i, $\cos\varphi_i$ is the power factor of substation i, v_i is the arriving power capacity of the load points supplied from substation i.

Therefore, the weighted load of substation i is the ratio of the arriving power and the power supply capacity.

Namely, suppose that η_{ij} is the power capacity supplying from substation i to load point j, $\gamma_{ij} = \frac{r_0}{U^2} l_{ij} \cos^2\varphi_i$ is the loss coefficient considerimg the distance from substation i to load point j, s_i is the transformer installing capacity of substation i and $\cos\varphi_i$ is the power factor, then the weighted load w_i of substation i is expressed as following.

$$w_i = \frac{\sum_{j=1}^{n} (\eta_{ij} - \gamma_{ij}\eta_{ij}^2)}{\alpha_i}, \qquad (1)$$

where U is the voltage, r_0 is the unit resistance, $\alpha_i = s_i r \cos\varphi_i$ is the power supply capacity of substation i.

In this time, the weighted load w_i changes as the power supply capacity η_{ij} changes.

Because the power need capacity of the loads changes as the time changes, the stable power distribution can be accomplished when the power supply range is determinated by the weighte d voronoi diagram.

2.2. The minimizing problem of power loss

When β_j is the power need capacity of substation j and $\bar{\xi}_{ij}$ is the arriving power capacity of substation i to load point j, the sum of the arriving power quantity to load point j must be equal to the power need capacity of that one. So

$$\sum_{i=1}^{m} \bar{\xi}_{ij} = \beta_j,\ \ j = \overline{1,\ n}$$

Now above expression changes as following under the condition $\bar{\xi}_{ij} = \beta_j \xi_{ij}$.

$$\sum_{i=1}^{m} \xi_{ij} = 1,\ \ j = \overline{1,\ n}$$

Because of $\eta_{ij} - \gamma_{ij}\eta_{ij}^2 = \beta_j \xi_{ij}$,

$$\eta_{ij} = \frac{1 \pm \sqrt{1 - 4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}}$$

Because of $4\gamma_{ij}\beta_j\xi_{ij} < 1$ and $2\gamma_{ij}\eta_{ij} < 1$ in practice,

$$\eta_{ij} = \frac{1 - \sqrt{1 - 4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}}.$$

Because the sum of start power capacity η_{ij} supplying from the substation i to the loads cannot be more than the power supply capacity of the substation i, the following inequality holds.

$$\sum_{j=1}^{n} \frac{1 - \sqrt{1 - 4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}} \le \alpha_i,\ \ i = \overline{1,\ m}$$

Therefore, the minimizing problem of the power loss by the weighted load on the power network is formalized as following.

$$\sum_{j=1}^{n} \frac{1 - \sqrt{1 - 4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}} \le \alpha_i,\ \ i = \overline{1,\ m}$$

$$\sum_{i=1}^{m} \xi_{ij} = 1,\ \ j = \overline{1,\ n}$$

$$\xi_{ij} \ge 0,\ \ i = \overline{1,\ m},\ \ j = \overline{1,\ n} \qquad (2)$$

$$\min_{1 \le i \le m} \left\{ \frac{\sum_{j=1}^{n} \beta_j \xi_{ij}}{\alpha_i} \right\} \Rightarrow \max$$

In (2), it can be changed as following by introducing the non negative variable ζ to the target function.

$$\min_{1 \le i \le m} \left\{ \frac{\sum_{j=1}^{n} \beta_j \xi_{ij}}{\alpha_i} \right\} \Rightarrow \max \Leftrightarrow \sum_{j=1}^{n} \beta_j \xi_{ij} \ge \alpha_i \zeta,\ i = \overline{1,\ m},\ \zeta \Rightarrow \max$$

Therefore, the problem (2) is transformed the folloing equivalent problem.

$$\sum_{j=1}^{n}\frac{1-\sqrt{1-4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}} \le \alpha_i,\ i=\overline{1,\ m}$$
$$\sum_{i=1}^{m}\xi_{ij}=1,\ j=\overline{1,\ n}$$
$$\sum_{j=1}^{n}\beta_j\xi_{ij}-\alpha_i\zeta \ge 0,\ i=\overline{1,\ m} \qquad (3)$$
$$\xi_{ij}\ge 0,\ i=\overline{1,\ m},\ j=\overline{1,\ n}$$
$$\zeta \ge 0$$
$$\zeta \Rightarrow \max$$

The problem (3) is the non linear program one which the target function is linear and has the convex bound.

In previous papers, the power supply radius of each substation was alphabetically determined by using the weighted voronoi diagram, and then solved the linear transtportation problem using the parametric method and determined the load point which is linked up with each substation by using this solution.

At this time, there is no guarantee that the power supply range with the maximum weighted load is determined and the optimical load points linked to each substation are collected because the loss quality of the power on the line isn't expressed in the linear function.

Therefore, the problems for evenly increasing the weighted load and for reducing the line power transmission loss must be discussed on one system in order to calculate the power supply range for minimizing the power loss.

2.3. The property of the problem for minimizing the power loss.

[Lemma 1] The following inequality is held at arbitrary point on the interval $[0,\ 1]$.

$$1-\sqrt{1-ax} \le (1-\sqrt{1-a})x,$$

where a is a given positive number which is smaller than 1.

(proof) $1-\sqrt{1-ax}=0$ and $(1-\sqrt{1-a})x=0$ if x=0 and $1-\sqrt{1-ax}=1-\sqrt{1-a}$ and $(1-\sqrt{1-a})x=1-\sqrt{1-a}$ if $x=1$.

Namely, $1-\sqrt{1-ax}=(1-\sqrt{1-a})x$ at the boundary of interval [0, 1]

About the function $y=1-\sqrt{1-ax}$, derivative of y is

$$y'=-\frac{1}{2}(1-ax)^{-\frac{1}{2}}(-a)=\frac{a}{2\sqrt{1-ax}}$$

and the secondary derivative of y is

$$y''=\left(\frac{a}{2\sqrt{1-ax}}\right)'=\frac{a}{2}\left(-\frac{1}{2}\right)(1-ax)^{-\frac{3}{2}}(-a)=\frac{a^2}{4\sqrt{(1-ax)^3}}$$

Because $0<a<1,\ 0\le x\le 1$, thus $0<a<1,\ 0\le x\le 1$.

Namely, because $0<a<1,\ 0\le x\le 1$ on the [0, 1], the function $y=1-\sqrt{1-ax}$ is the strict convex one passing through the points (0, 0) and $(1,\ 1\text{-}\sqrt{1-a})$.

And because the function $(1-\sqrt{1-a})x$ is the linear one passing through the points (0, 0) and $(1,\ 1\text{-}\sqrt{1-a})$, the following enequality is held at the arbitray point of the interval [0, 1].

$$1-\sqrt{1-ax}\le(1-\sqrt{1-a})x$$

Especially, at the arbitray point of the interval (0, 1),

$$1-\sqrt{1-ax}<(1-\sqrt{1-a})x.$$

(end of proof)

From (1), following relationship between the start power supply capacity η_{ij} and the arriving power $\bar{\xi}_{ij}$ of substation i to load point j holds.

$$\bar{\xi}_{ij}=\eta_{ij}-\gamma_{ij}\eta_{ij}^2 \Leftrightarrow \beta_j\xi_{ij}=\eta_{ij}-\gamma_{ij}\eta_{ij}^2$$

Because the loss coefficient $\gamma_{ij}(>0)$ is small as compared with β_j, we can say $4\gamma_{ij}\beta_j\le 1$ at the arbitrary i and j,no losing generality.

Therefore, the corollary is following from lemma 1

[Corollary 1] At arbitrary point of the interval [0,1],

$$\frac{1-\sqrt{1-4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}}\le\frac{1-\sqrt{1-4\gamma_{ij}\beta_j}}{2\gamma_{ij}}\xi_{ij} \qquad (4)$$

Especially, the inequality (4) changes to the following equality if ξ_{ij} is equal to 0 or 1.

$$\frac{1-\sqrt{1-4\gamma_{ij}\beta_j\xi_{ij}}}{2\gamma_{ij}}=\frac{1-\sqrt{1-4\gamma_{ij}\beta_j}}{2\gamma_{ij}}\xi_{ij}$$

Now, if we suppose that

$$\theta_{ij}=\frac{1-\sqrt{1-4\gamma_{ij}\beta_j}}{2\gamma_{ij}},\ i=\overline{1,\ m},\ j=\overline{1,\ n},$$

then the problem (3) can be relaxed to the following linear programming.

$$\sum_{j=1}^{n}\theta_{ij}\xi_{ij} \le \alpha_i,\ i=\overline{1,\ m}$$
$$\sum_{i=1}^{m}\xi_{ij} = 1,\ j=\overline{1,\ n}$$
$$\sum_{j=1}^{n}\beta_j\xi_{ij} - \alpha_i\zeta \ge 0,\ i=\overline{1,\ m} \qquad (5)$$
$$\xi_{ij} \ge 0,\ i=\overline{1,\ m},\ j=\overline{1,\ n}$$
$$\zeta \ge 0$$
$$\zeta \Rightarrow \max.$$

If the allowable range of problem (5) isn't empty set, there must be an optimical vector because it is the bounded closed set.

[Lemma 2] If the allowable range of the problem (5) isn't empty set, the optimical value of (5) is the low bound of the one (3).

(proof) Suppose that the allowable range of (5) isn't empty set and the allowable ranges of (3) and (5) are individually $S_1,\ S_2$.

About $x \in S_2$, $S_2 \subset S_1$ because $x \in S_1$ from the inequality (4).

Therefore,

$S_1 \ne \phi,\ \max\{\zeta \mid x \in S_2\} \le \max\{\zeta \mid x \in S_1\}$

Namely, the optimal value of the problem (5) is the low bound of one of (3). (end of proof)

[Lemma 3] If the allowable range of problem (5) isn't empty set and the optimal vector of (5) is 0-1 vector, then the vector is the optimal one of (3).

(proof) Because the target functions of the problem (5) and (3)about 0-1 vector of the allowable range are equal, these two problems are equivalent about 0-1vector.(end of proof)

Let's $\theta_j = \min_{1\le i\le m}\{\theta_{ij}\},\ j=\overline{1,\ n}$.

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[Lemma 4] About an arbitrary allowing vector of problem (5), $\sum_{j=1}^{n}\theta_j \le \sum_{i=1}^{m}\alpha_i$.

(proof) Let's $x=(\xi_{ij})_{m,n}$ is an arbitrary allowing vector of problem (5).

Then because $\sum_{i=1}^{m}\xi_{ij}=1,\ j=\overline{1,\ n}$,

$$\sum_{i=1}^{m}\alpha_i \ \ge\ \sum_{i=1}^{m}\sum_{j=1}^{n}\theta_{ij}\xi_{ij} \ =\ \sum_{j=1}^{n}\sum_{i=1}^{m}\theta_{ij}\xi_{ij}$$
$$\ge\ \sum_{j=1}^{n}\sum_{i=1}^{m}\theta_j\xi_{ij} \ =\ \sum_{j=1}^{n}\theta_j\sum_{i=1}^{m}\xi_{ij} \ =\ \sum_{j=1}^{n}\theta_j$$

Namely, $\sum_{j=1}^{n}\theta_j \le \sum_{i=1}^{m}\alpha_i$. (end of proof)

Because $0 \le \xi_{ij} \le 1,\ i=\overline{1,\ m},\ j=\overline{1,\ n}$, the allowing range of (5) is empty or the bounded closed set.

Let's $\bar\theta_j = \max_{1\le i\le m}\{\theta_{ij}\},\ j=\overline{1,\ n}$.

[Theorem 1] If $\sum_{j=1}^{n}\bar\theta_j \le \sum_{i=1}^{m}\alpha_i$, then the allowing range of (5) is not empty set.

(proof) Let's $\xi_{ij}=\frac{\alpha_i}{a},\ i=\overline{1,\ m},\ j=\overline{1,\ n}$, where $a=\sum_{k=1}^{m}\alpha_k$.

About an arbitrary $i\ (1\le i\le m)$,

$$\sum_{j=1}^{n}\theta_{ij}\xi_{ij} = \sum_{j=1}^{n}\theta_{ij}\frac{\alpha_i}{a} = \alpha_i\frac{\sum_{j=1}^{n}\theta_{ij}}{a} \le \alpha_i\frac{\sum_{j=1}^{n}\bar\theta_j}{a}$$

At this time, $\sum_{j=1}^{n}\bar\theta_j \le \sum_{i=1}^{m}\alpha_i$.

Therefore, $\alpha_i\frac{\sum_{j=1}^{n}\bar\theta_j}{a} \le \alpha_i$

.

Namely, $\sum_{j=1}^{n}\theta_{ij}\xi_{ij} \le \alpha_i$.

And about an arbitrary $j\ (1\le j\le n)$,

$$\sum_{i=1}^{m}\xi_{ij} = \sum_{i=1}^{m}\frac{\alpha_i}{a} = \frac{\sum_{i=1}^{m}\alpha_i}{a} = 1.$$

Clearly, $\xi_{ij}\ge 0,\ i=\overline{1,\ m},\ j=\overline{1,\ n}$, so $(\xi_{ij})_{m,n}$ is the allowing vector of (5). And about an arbitrary non negative $(\xi_{ij})_{m,n}$, there is a non negative number ζ .(end of proof)

Soppose that $x^*=(\xi_{ij}^*)$ is the optimical vector of (3) and let's

$$\theta_{ij}^* = \frac{1-\sqrt{1-4\gamma_{ij}\beta_j\xi_{ij}^*}}{2\gamma_{ij}}, \ i=\overline{1,\ m},\ \ j=\overline{1,\ n}\ . \quad (6)$$

If $0<\xi_{ij}^*<1$ about any load point *j*, that is supplied thepower from the several substations(for example, from substations $i_1,\ i_2,\ \cdots i_k$)) and the following problem can be discussed, in which the number of the load points is increased to n_1 when this substation *j* is separated to *k* load points with the need powers $\beta_j\xi_{i_k j}^*$.

$$\begin{aligned}
&\sum_{j=1}^{n_1}\theta_{ij}^*\xi_{ij} \le \alpha_i,\quad i=\overline{1,\ m}\\
&\sum_{i=1}^{m}\xi_{ij} = 1,\quad j=\overline{1,\ n_1}\\
&\sum_{j=1}^{n_1}\beta_j\xi_{ij} - \alpha_i\zeta \ge 0,\quad i=\overline{1,\ m}\\
&\xi_{ij}\ge 0,\quad i=\overline{1,\ m},\quad j=\overline{1,\ n_1}\\
&\zeta \ge 0\\
&\zeta \Rightarrow \max\ .
\end{aligned}\qquad (7)$$

[Theorem 2] If the allowable range of (7) is not empty, the optimal vector of (3) is the one of (7).

(Proof) From the proving process of lemma 2, the arbitrary allowable vector of (7) is one of (3) and the target functions of these are equal.

Now suppose that the optical vector $x^*=(\xi_{ij}^*)$ of (3) is not one of (7).

Then there is an allowable vector $\bar{x}=(\bar{\xi}_{ij})$ of (7), the allowable value $\bar{\zeta}$ of (7) is bigger than the optimal value ζ^* of (3).

At this time, $\bar{x}=(\bar{\xi}_{ij})$ is the allowable vector of (3) and the value of target function is $\bar{\zeta}$.

This contradicts the truth that $x^*=(\xi_{ij}^*)$ is the optimal vector of (3). (end of proof)

Let's discuss the following problem when $x_0=(\xi_{ij}^0)$ is the optimal vector of (5) and

$$\theta_{ij}^0 = \frac{1-\sqrt{1-4\gamma_{ij}\beta_j\xi_{ij}^0}}{2\gamma_{ij}},\ i=\overline{1,\ m},\ \ j=\overline{1,\ n}\ .$$

$$\begin{aligned}
&\sum_{j=1}^{n}\theta_{ij}^0\xi_{ij} \le \alpha_i,\ i=\overline{1,\ m}\\
&\sum_{i=1}^{m}\xi_{ij} = 1,\ \ j=\overline{1,\ n_1}\\
&\sum_{j=1}^{n}\beta_j\xi_{ij} - \alpha_i\zeta \ge 0,\ i=\overline{1,\ m}\\
&\xi_{ij}\ge 0,\ i=\overline{1,\ m},\ \ j=\overline{1,\ n_1}\\
&\zeta\ge 0\\
&\zeta\Rightarrow \max.
\end{aligned}\qquad ,\quad (8)$$

where n_1 means the same as (7)

[lemma 5] The allowable range of (8) is not empty.

(proof) For the arbitrary non negative number μ and λ of the interval [0, 1],

$1-\sqrt{1-\mu\lambda}\le(1-\sqrt{1-\mu})\lambda$.

So

$\theta_{ij}^0\le\theta_{ij},\ i=\overline{1,\ m},\ \ j=\overline{1,\ n}$.

Therefore, the allowable range of (5) belongs to the one of (8) and the allowable range of (8) is not empty set because of the existence of the optimal vector of (5). (end of proof)

If the optimal vector $x_0=(\xi_{ij}^0)$ of (5) is determined, the power supply range of each substation is determined as following.

$V(w_i)=\{j\in N\mid \xi_{ij}^0=1\},\ i\in M$

At this time the power supply radius of each substation is determined as following.

$L_i=\max\{l_{ij}\mid j\in V(w_i)\},\ i\in M$

4). The minimizing problem of power loss on the radial network.

The radial network needs that each load point must be supplied from only one substation.

Therefore, the minimizing problem of power loss in radial network is one which variable *x* of (5) needs to be 0-1 vector .

Namely, the minimizing problem of power loss in radial network is formlized the following mixed 0-1 linear programming.

$$\begin{aligned}
&\sum_{j=1}^{n}\theta_{ij}\xi_{ij} \le \alpha_i,\ i=\overline{1,\ m}\\
&\sum_{i=1}^{m}\xi_{ij}=1,\ j=\overline{1,\ n}\\
&\sum_{j=1}^{n}\beta_j\xi_{ij}-\alpha_i\zeta \ge 0,\ i=\overline{1,\ m} \qquad\qquad (9)\\
&\xi_{ij}\in\{0,\ 1\},\ i=\overline{1,\ m},\ j=\overline{1,\ n}\\
&\zeta \ge 0\\
&\zeta \Rightarrow \max.
\end{aligned}$$

If $\alpha_i,\ \beta_j$ are the positive integers in the problem (9), $\alpha_i\zeta$ can be also selected as non negative integer, so (9) becomes to all integer linear programming.

In reality, $\alpha_i,\ \beta_j$ are all the positive integers.

In reality, a load point can be supplied from the several substations because of its speciality.

The consecutive problem of (9), that is, the following problem is the first step one for minimizing the power loss in radial network.

$$\begin{aligned}
&\sum_{j=1}^{n}\theta_{ij}\xi_{ij} \le \alpha_i,\ i=\overline{1,\ m}\\
&\sum_{i=1}^{m}\xi_{ij}=1,\ j=\overline{1,\ n}\\
&\sum_{j=1}^{n}\beta_j\xi_{ij}-\alpha_i\zeta \ge 0,\ i=\overline{1,\ m} \qquad\qquad (10)\\
&\xi_{ij}\ge 0,\ i=\overline{1,\ m},\ j=\overline{1,\ n}\\
&\zeta \ge 0\\
&\zeta \Rightarrow \max.
\end{aligned}$$

(10) is the linear programming problem and there must be the optimal vector if its allowable range is not empty.

At this time the optimal vector of (10) can be generally expressed as an convex polyhedron.

[attention] Because the rank of the coefficient matrix of the linear transport problem is $m+n-1$, so the rank of the coefficient matrix of the first step problem of minimizing the power loss on the radial network is not bigger than $m+n-1$.

Therefore, at the most $m-1$ load points are supplied from the substations of more than 2.

Let's S is the set of the optimal vectors of (10) and discuss the second step problem of minimizing the power loss.

$$x=(\xi_{ij})_{m,n}\in S,\ \sum_{i=1}^{m}\sum_{j=1}^{n}(\xi_{ij}^2-\xi_{ij})\Rightarrow \max. \qquad (11)$$

The problem (11) is the secondary concave programming one.

The optimal vector $x^0=(\xi_{ij}^0)$ of this problem can evenly increase the weighted load and also minimize the cost for line moving.

III. THE SOLVING METHOD OF MINIMIZING THE POWER LOSS IN RADIAL NETWORK

In reality, because the power need quatity is not smaller than the power supply capacity which the average loss coefficient is considered, we can first selecte an suitable positive number (proportion coefficient) which is bigger than 1 and use the value that multiplies it to power supply ability of each substation as the new supply capacity, so that the allowing range of (9) exists.

Therefore, without loss of generality we can suppose that the allowing range of (9) certainly exists.

Step 1. Sellect the suitable proportion coefficient p and renew the supply ability of each substation.

Step 2. Calculate the optimical point of the moving restrict linear programming problem, the continuous one of (9) which replaces $0\le\xi_{ij}\le 1$ instead of $\xi_{ij}\in\{0,\ 1\}$ by using the group method.

Step 3. If the x- optimal vector of consecutive problem is 0-1 vector, it is the optimal point for minimizing the power loss on the radial network.

Go to step 6.

If the x- optimal vector is not 0-1 vector, go to next step.

Step 4. Discuss the concave quadric programming which replace the target function to

$$f(x,\zeta)=\zeta+\mu\sum_{i=1}^{m}\sum_{j=1}^{n}(\xi_{ij}^2-\xi_{ij})\Rightarrow \max.$$

and find the local extremal vertex by using the group method and cutting one.

Find the valid section linear inequality in order to determine the range, the local extreme vertex of which becomes to the optical point.

For about the normal basic system of solutions

$d_i=\begin{pmatrix}x_i\\0\end{pmatrix}, i=\overline{1,k}\quad d_{k+1}=\begin{pmatrix}x_{k+1}\\1\end{pmatrix}$, if x_{k+1} is the local extreme vertex, the section inequality is $\sum_{i=1}^{k}\frac{\xi_i}{2\lambda_i}\ge 1$, where

$$2\lambda_i=\frac{-\nabla f(x_{k+1})^{\mathrm{T}}x_i}{x_i^{\mathrm{T}}Ix_i}.$$

Sever the allowable range by the valid section inequality and go to next step if the allowable range is empty or go to step 2 if its't empty.

Step 5. Find the x—part of the 0-1 vector among the given local extreme vertexs, then the vector which ζ has the maximum value of them is the optimal point for minimizing the power loss.

Goto step 6.

Step 6. By using the above optimal vector, calculate the power supply capacity $\overline{\alpha}_i$ of each substation and determine the transform coefficients $\delta_i = \alpha_i / \overline{\alpha}_i$.

Renew the need capacities of the load points which are supplied from the substation i to the values which are multiplied the δ_i to the original need qualities.

Namely, though the load point j needs β_j, only $\delta_i \beta_j$ is supplied because of the power loss.

IV. APPLIED EXAMPLES

[Example 1] We analyzed the reality of the power distribution network in the region A and calculated the weighted loads of the substations and the power loss quantity based on the above model.

Now the power distribution network of region A consists of the 10 substations and 507 load points, and it is a radial network.

The power loss rate of the region A is calculated as table 1.

Table 1. Power loss rate of the existing power network (power factor=0.88, voltage=3kV)

substation	Equivalent resistance	Average power supply radius (km)	Power loss ratio(%)
1	0.004	6.69	25.7
2	0.01232	6.94	11.48
3	0.00415	5.11	19.1
4	0.00838	8.87	19.6
5	0.00678	6.06	17.75
6	0.00206	3.9	15.14
7	0.00159	5.15	14.71
8	0.01316	15.9	20.61
9	0.00334	1.72	13.63
10	0.0045	8.23	12.21

From the table 1, we can see that the power loss rate of the substations 8 and 1 isn't smaller than 20 and existing average loss rate of this region is relatively big as 16.99%.

Based on the above model, we constructed the radial network by separating the power supply range of the substations.

The reault of calculating the power loss rate is shown in the table 2.

Table 2. The power loss ratio of newly constructed radial power distribution network (power factor=0.88, voltage=3kV)

substation	Supply capacity (kw)	Average power Supply range (km)	Power loss rate(%)	Weighted load
1	3400	5.35	5.78	0.84
2	2300	3.54	17.88	0.72
3	2500	5.34	14.67	0.76
4	2000	2.71	13.66	0.75
5	3700	7.86	18.79	0.72
6	2800	7.59	6.86	0.81
7	1700	8.2	4.69	0.82
8	1400	11.38	21.32	0.78
9	2100	6.62	17.65	0.73
10	3100	4.68	12.3	0.77

From the table 2, the power loss ratio of new radial network is not bigger than 21% and the regional average loss ratio is 13.36%, which can decrease the loss ratio 3.33% than the existing network.

And the regional average power supply radius is 6.33km, which is shorter about 0.53km than the existing network.

In this power network the minimum weighted load is also 0.73, maximum one is 0.84 and average one is 0.79, so this power network can satisfy the need power of the load points at more than 79%.

As the result of calculation, the supply capacity of the substations in region A is smaller than the need quantity of the load points, so new substations must be constructed and we can know that the reasonable power distribution network can be constructed so that the ability of substations balances with need quantity of load points and eliminate the power loss.

[Example 2]

We also analyzed the power distribution network of region B.

We determined the weighted loads of the substations and calculated the power loss quantities by using the above model and method.

Now the power distribution network of the region B consists of 6 substations and 428 load points and it is the radial network.

Though the power supply radius of the existing network is smaller than the one of new method, but some load points cannot be supplied the power.(7% of the load points linked to the substation 1, 4% of the ones linked to the substation 4 and 16.9% of ones linked to the substation 5,etc)

As the result of calculation, we can see that the reasonable power distribution network must be reconstructed so that the supply capacity of the substations in region B can satisfy the need quantity of load points and eliminate the power loss.

Table 3 the power distribution network calculation data of the region B (power factor=0.85, voltage=10kV)

substation	supply capacity (kw)	Supply quantity (kw)	Loss quantity (kw)	Arriving quantity (kw)
1	4250	4189.7	800.58	3389.12
2	10795	9413.31	804.94	8608.37
3	11220	9581.57	634.29	8947.28
4	5610	4936.83	463.19	4473.64
5	8075	6644.09	204.76	6439.33
6	8500	6988.13	209.88	6778.25
substation	Weighted load	Power loss ratio (%)	Supply radius (m)	
1	0.797	19.11	3297.18	
2	0.797	8.55	5033.26	
3	0.797	6.62	5122.75	
4	0.797	9.38	4671.96	
5	0.797	3.08	6731.78	
6	0.797	3	5410.98	

In the new network, the average power loss rate is 8.29%, so we can eliminate more 2% than the existing network and supply enough the power need quantity of every load points with the existing supply capacity of substations in region B.

The comparison with the traditional method is shown in the table 4.

Table 4. comparison with the traditional method

substation	supply capacity (kw)	Supply quantity (kw)	need quantity (kw)	Power loss ratio (%)
1	4250	4546.65	4030	12.82
2	10795	8529.85	7765	9.85
3	11220	11077.52	10175	8.87
4	5610	5834.01	5156	13.15
5	8075	9442.89	8720	8.29
6	8500	3047.52	2790	9.23
substation	Supply radius (m)	deviation (kw)	Load points which cannot be supplied %)	
1	2065.74	296.7	7	
2	2225.54			
3	1789.52			
4	5378.79	224	4	
5	3444.6	1367.9	16.9	
6	1481.74			

Now the average power loss rate of existing power distribution network of the region B is 10.36% and the power supply capacity can satisfy the need quantity of the load points, but every load points cannot be supplied enough power because the reasonable power distribution network wasn't constructed.

V. CONCLUSION

First, We proposed an mathematical model and researched the properties which can evenly increase the weighted load of the substations and eliminate the power loss.

Second,We researched the calculating method to get the optimal solution for minimizing the power loss on the radial network.

Third, We tested the effectiveness of this method in several regions.

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